

LEARNING APPROACHES
IN
MATHEMATICS

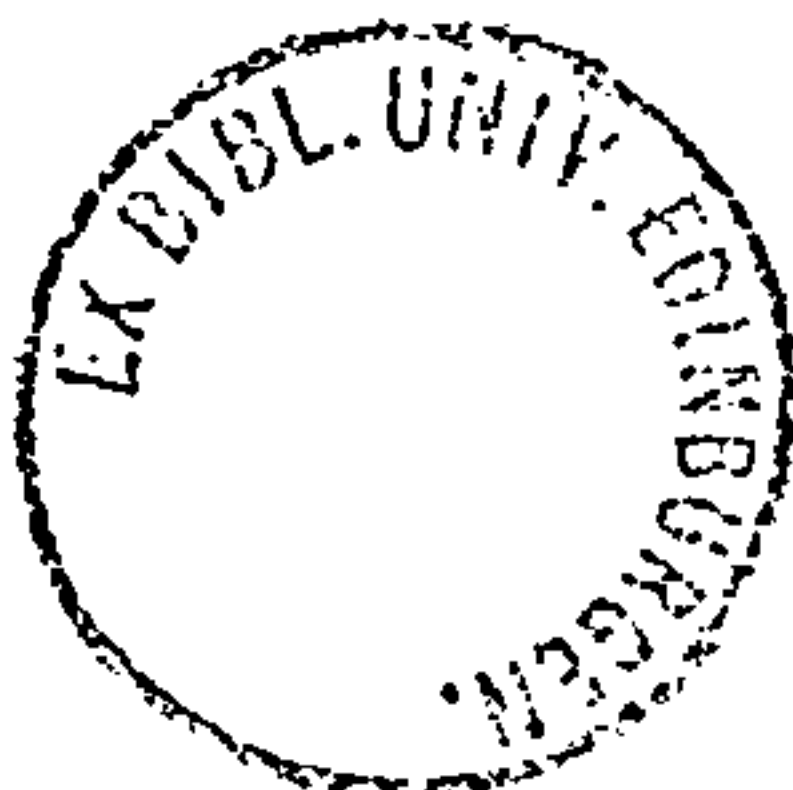
by

DESPINA POTARI

Ph.D. Thesis

Department of Mathematics
Faculty of Science
UNIVERSITY OF EDINBURGH

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ABSTRACT

The introduction of the Action Plan in 1983 produced a number of changes in non-advanced post compulsory education in Scotland. New assessment procedures were introduced together with a modular course structure.

In Chapter one, the educational features of the new system and particularly those of the SOCTVEC mathematics module Mathematics 2/Analysis 1 (M_2/A_1) are discussed.

Chapter two deals with the development of teaching/learning materials for that Mathematics module.

In Chapter three the evaluation of these materials and their role in the learning environment are described.

In Chapter four the implications of the study are discussed.

PREFACE

The research described in this thesis was conducted under the supervision of Dr. J.W. Searl, Department of Mathematics, and Mr. A.B. Pollitt, Department of Education.

Declaration

This thesis is my own work. I have not submitted it, or any part of it, in a previous application for a degree.

..........
(Despina Potari)

April 1987

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of 40 hours. Part-time students can be employees or apprentices with day-release, pupils in schools who attend for less than the full-school week, adults who take a limited number of subjects.

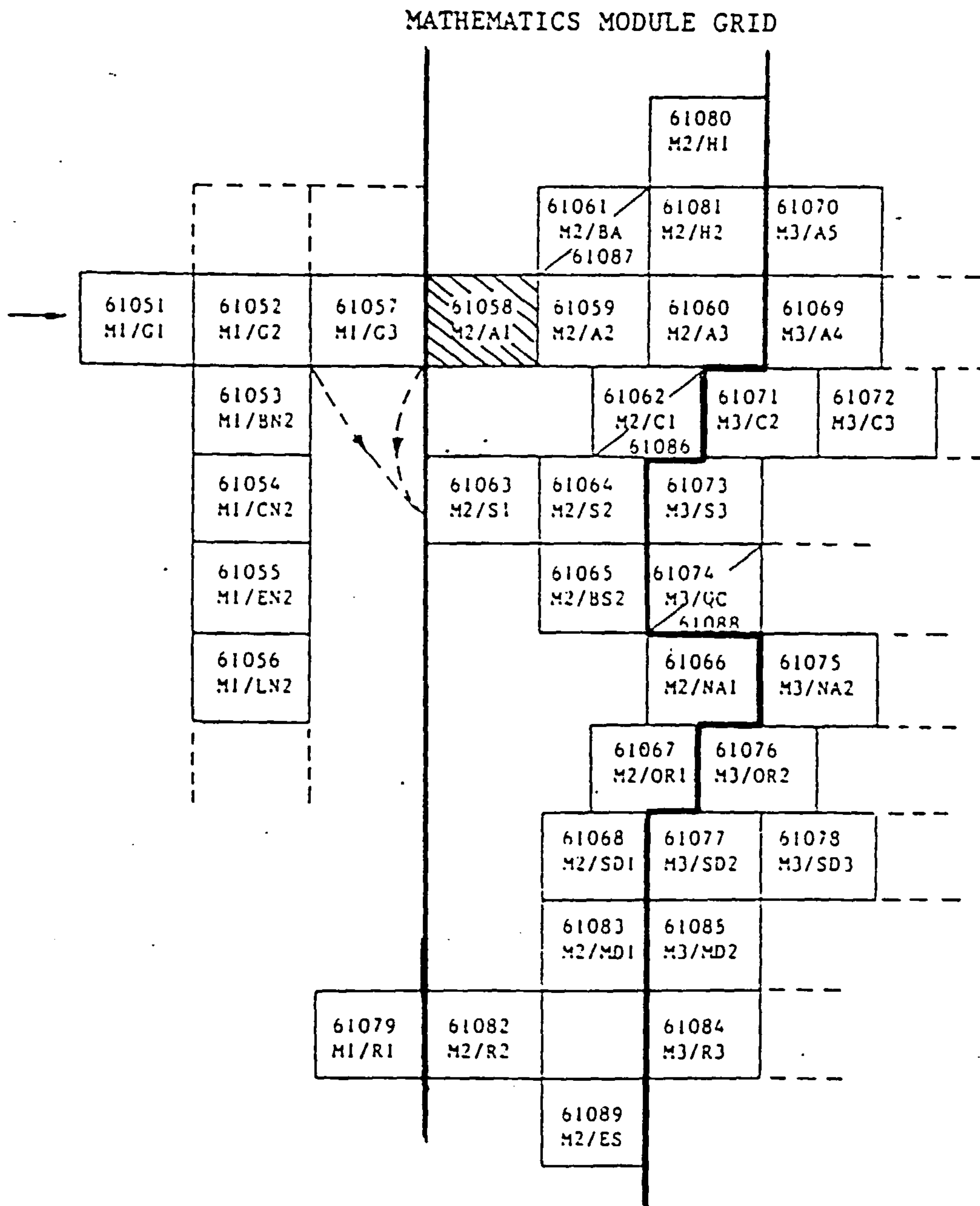
In January 1984, the first module descriptors were produced by development teams which had been formed after the publication of the Action Plan. By August 1986 about 2000 modules have become available. There are two main types of modules: general and specialist. In the module descriptors a preferred entry level, a set of learning outcomes which specify what the student should know, a content/context description, learning/teaching approaches and assessment procedures are described. They were published by the Scottish Vocational Education Council (SCOTVEC 1986).

In figure 1.1 the mathematics module grid is shown. The grid shows the structure of the mathematics modules and some of their relationships. The groups of modules with the prefix M_1 correspond with the range of mathematics which the students have encountered at school before the age of 16. Some of the modules with prefix M_2 correspond approximately with the mathematical content of Higher Course. The groups of modules with the prefix M_3 are classified as post-Higher mathematics.

Most of the modules are offered as 40-hour modules. $M_1|G_2$, $M_1|G_3$ can also be offered as 80-hour modules. Modules which are displayed with a diagonal in the middle grid have a flexibility in the time location.

Modules A_1, A_2, A_3, C_1 cover the Higher mathematics syllabus but alternative groupings such as A_1, A_2, C_1 and S_2 are possible for students desiring to continue with Higher Education (college or university studies).

Figure 1.1



This study is concerned with the mathematics module Analysis | Algebra 1 ($M_2|A_1$). It is a general module which is designed to present mathematical concepts within the context of applications. Students who have O-Grade mathematics C pass, Standard Grade mathematics at level 3 or some qualification equivalent to this level, for example Mathematics module $M_1|G_3$ can enter this module. It is a key module, especially for those who wish to study mathematics further. The content of this module is about one third of the Higher mathematics syllabus and of the Revised Higher (Scottish Examination Board, 1986).

In the Appendix A the latest version of the module descriptor for $M_2|A_1$ (1986-87) is shown. This is almost the same as the one published by SCOTVEC in Spring 1984. There are some small differences in the content (for example, 3-dimensional vectors were not included in the first version). In the learning and teaching approaches, computer access, group problem solving and practical investigations are regarded as essential parts of the module. Moreover, the use of self-help remedial materials is suggested to reinforce skills.

To satisfy the learning and teaching approaches of the module, teachers have to produce or share teaching/learning materials in order to cover the different needs of students. In the Action Plan, it has been seen that teachers "should allow more time for preparation of course materials and development of appropriate teaching methods." This suggestion has not seemed to work during the first years of the implementation of modules (1984-86). The main reason is that teachers have many responsibilities: they are assessors and counsellors as well as tutors. In addition they have to adopt a role different from that they used to, having to achieve the flexibility proposed in the Action Plan. For these reasons they do not have the time to produce teaching/learning materials which are essential to their course.

Previous work for the module $M_2|A_1$.

The development teams who worked on the Mathematics modules produced, in a very short period, guidelines on teaching and learning approaches and on the assessment of the modules. They produced guidelines on modules $M_1|G_2$ and $M_2|A_1$ regarding them as exemplar modules for elementary (M_1) and advanced (M_2 and M_3) respectively.

In the guidelines for teaching and learning approaches a learning environment is suggested using the essential ingredients which have been identified in the Cockcroft Report, paragraph 243. Teachers, fellow and senior students, experience, books, television, films, formal self-instructional materials, posters, exhibitions, are also regarded as essential. Learning resources which have been suggested are practical activities, investigations, books, tape/booklet sequences, video-tape sequences, computer programs and posters. The materials which had not been evaluated are summarised below.

Practical activities: The desirability of having a variety of different activities is emphasized to meet the needs and interests of all the students. The guidelines point out that help and direction will be necessary for the students as they are not used to this learning approach. Fourteen activities are described in the guidelines, from one to four items for every mathematical topic of the module.

Errors: Two activities on measurement and errors are described, one about large measurements and the other about small.

Linear Functions: Four activities are described, three about fitting a straight line to experimental data and the fourth about the concept of gradient.

Quadratic Functions: Three activities are described; two emphasize the relationship of the formula of the quadratic function and its graph and the third is about the solution of a quadratic equation by the use of a device.

Looking for an experimental law: An experiment is described and the experimental law is sought by plotting the set of experimental points.

Trigonometric Functions: Three activities are described. One is about the drawing of sine curves from a number of experimental points, another about the use of trigonometry for calculating heights of buildings and the third one involves plotting sine-cosine curves using a mechanical analogue.

Vectors: An activity about the addition of vectors using three spring balances is described.

Investigations: It is emphasized that the investigations must be for all the students not only for the more able and that they will need help with this work. A written essay in which they describe their work on the investigation is suggested.

Four investigations are suggested: the design of a milk carton, a book based investigation about properties of parabola (Lockwood, 1978) an old puzzle about the calculation of the height of an arc supported by a chord of half a mile and of length of one foot longer than the chord, and an investigation about vector algebra using magic squares.

Books: Six series of books are suggested.

Tape/booklet sequences: These sequences consist of booklets with audio-tape commentaries which take the student through the mathematics in a step-by-step manner, each step requiring a response. An example on Error Algebra is described in the guidelines.

Mathematics on Videotape: Videotaped solutions to a selection of problems have been produced. Two examples are described in the guidelines. The speaker works step-by-step on the problem using informal language and showing the process of his thinking. These tapes can be used by individuals or by small groups.

Computer aided learning: Seven simple computer programs written in Basic for a BBC Model B are described.

ERROR: It calculates the speed of a vehicle when the distance and time are given and encourages the students to look critically at the results.

GRAPH: The students are asked to estimate the point where the minimum of a curve occurs. The iteration stops at ten decimal places.

LINEAR: It fits a straight line to a given set of co-ordinates using the method of least squares.

QUAD: It produces the graph of the quadratic function $f(x) = ax^2 + bx + c$ if the student supplies a, b, c . Only values $0 < x < 10$ and $-100 < y < 100$ are plotted on the screen.

PARABola: It simulates a penny being 'flicked' from a table top. It displays the velocity required to land the penny on that spot and the equation of the path given the x -value.

SIN: It draws functions of the form $y = a \sin bx$ where x is in radians.

FENCE: It draws a fence post with a single wire attached. Given the angle of the second wire and the respective tensions the drawing is completed. The vector diagram and the resultant are also shown.

Posters: Posters can provide background information, stimulate interest by presenting problems or puzzles and encourage good study skills. Some examples of posters are given.

In the guidelines for the assessment procedures criterion-referenced formative and summative assessment methods are described.

The formative assessment consists of the student workfile where the students keep all their work during the module and of diagnostic worksheets. The worksheets seek to determine whether the student is coping with the current work of the class.

The summative assessment consists of a short answer paper, an extended answer paper and the student's workfile. A score of 45 out of 60 is regarded as satisfactory for the short answer paper and 15 out of 40 for the extended answer paper. Students whose performance is not satisfactory are asked to resit the test where they failed after some remedial work.

The guidelines suggest roughly eight weeks for covering the content of the module, followed by two weeks for revision and consolidation. Each week covers four hours of the 40 hours module. It is suggested that one week can be devoted to error algebra and its applications, one week to graphs and practical modelling, one week to linear functions, two weeks to quadratic functions, two weeks to circular functions and one week to vectors. In figure 1.2 a student progress report shows the distribution of the content of the module in eight weeks. It also shows the assessment requirements.

Figure 1.2

STUDENT PROGRESS REPORT

MODULE M2/A1

Name: _____

College/School: _____

FORMATIVE	Outcome 1	Measurement/Error Sc.Notation/Rounding Error/Error Bounds Applications Workfile
	Outcome 2	Pract. Modelling Graphs Sub/Eval/Error Linear Interpolat. Linear Models Linear Equations Transposition Applications Workfile Quadratic Models Graphs Equations Sub/Eval/Errors Area Sq.Root Function Transposition Applications Workfile Ratios Graphs Problems Area Radians Triangles Sub/Eval/Errors Applications
		Linear
		Quadratic
FINAL ASSESSMENT	Outcome 3	Diagrams Components Position/Unit Applications
	Test	SECTION 1 SECTION 2
		Workfile

Module satisfactorily
completed:
(Signed) _____

(Status) . _____

(date) _____

A = outstanding, S = satisfactory, R = resit, resubmit
or take remedial action

About the study

This study is concerned with the development of learning/teaching materials for the mathematics module $M_2|A_1$. It aims to see if a learning environment of the type suggested by the guidelines can be created and to evaluate the individual components as well as the overall environment. It might also help teachers to modify their teaching roles in the directions suggested by the guidelines. In developing the materials the intention has been to meet the needs of all the students. Each student could choose from the whole suite of the materials, with the help of his/her teacher, and select those which meet his/her learning patterns and needs.

The learning/teaching materials include computer programs, videotaped expositions, consolidation and practice exercises, practical activities, tape/booklet sequences, posters and a collection of investigations. These materials are described in Chapter 2.

The development of the materials started in January 1985, two years after the publication of the Action Plan. The materials have been tested in colleges of Further Education and schools in Scotland. The evaluation adopted the illuminative approach (Malcolm Parlett, David Hamilton, 1972) and consisted of two stages. One stage was intended to evaluate the individual materials while the second was intended to show how the materials fit in the whole learning environment. The procedures of evaluation are described in Chapter 3.

CHAPTER TWO

DEVELOPMENT OF LEARNING/TEACHING MATERIALS

Computer aided learning materials

Computer aided learning is recommended in both the guidelines and the module descriptor for $M_2|A_1$.

The vast technological change and the needs of the society require people, nowadays, to be computer literate. "Today's schoolchildren were born in the "computer age" and must leave school as computer-literate adults if they are to function in a computer-oriented society" (Glass, 1984). Moreover, rightly used, computers can improve the learning process in mathematics. The reasons for this lie in the capabilities of computers, especially microcomputers. They can provide interactive graphic displays which allow mathematical concepts to be presented more efficiently than in a textbook. Animation can also help to clarify higher-level concepts. They can perform calculations very quickly. "Microcomputers are powerful means of doing mathematics extremely quickly and sometimes in a visually dramatic way" (Curriculum matters 3, 1985). The computer can provide an immediate and flexible response to the students' work and opportunities for them to try their own examples and to investigate mathematical ideas. It can also provide a genuinely individualised learning resource and so provide stronger motivation to the students.

Positive contributions of computers to the learning process, however, require suitable computer programs. Mackie (1986) argues that the use of computers can enhance a student's mathematical experience but to maximise this experience use of

appropriate well designed software is needed. Micro-computers are still expensive devices compared with other media. In addition to this, the development of computer programs requires effort and time, so careful thought is demanded about the design of computer programs.

There is no doubt that the computer can provide motivation. Some believe it to be a halo effect and that children will become tired of the medium. Even if this were true, it would merely mean that computer assisted learning should be used economically where it can give most benefit as for all teaching techniques.

(Beveridge, 1984)

The computer aided learning must offer something more than 'drill and practice', it must enhance students' understanding of mathematical concepts. Bajpai et al., (1985) believe that computer programs should provide opportunity for discovery, interaction between program and user, good presentation for example colour, sound, graphics, to enhance students' understanding. Phillips (1986) also emphasizes the care needed in using suitably the graphics capabilities of the computer. Phillips suggests that the exploitation of the diagram can only be through the discovery of new ways of presenting ideas and new ways of thinking.

In this study eight computer programs have been developed to complement ordinary class teaching. They are short, to avoid consuming too much of the students' time, and present mathematical concepts encountered by the students during the module. The first seven programs aim to make one teaching point each, although inevitably there is some incidental teaching within each program depending on the knowledge and skills of the student. The eighth

program is of an integrative nature drawing ideas from several parts of the module. The teaching points of the programs are described in Table 2.1.

Table 2.1

Program	Teaching Point(s)
Errors	How to find a sensible answer for the quotient given by a computer or a hand calculator.
Minimum	Location of the minimum value of a function from its graph by trial and error.
Line	Fitting a straight line to experimental data and its reliability.
Quadratic function	The shape of the graph of the quadratic function and its properties.
Cosine-Sine curves	Properties of the cosine graph and approximations to periodic functions.
Radian	Relationship between radians and degrees.
Vectors	Addition of vectors.
Bridge	Shape of parabola, lines of action of balanced forces meeting in a point, resolution and evaluation of forces, rounding of numbers.

The way in which the programs are used is left to the teacher although some suggestions are given in the teaching guide for the computer programs (Appendix D). The programs may be used as illustrative materials during a 'chalk and talk' exposition, or as part of the students' work or as remediation exercises.

Each program comes with an appropriate worksheet for the student to complete for his/her workfile (see Appendix D). On the worksheet, instructions are given to the student about the procedure needed to see the program. A first example is suggested for the student to try in most of the worksheets, so that the student who has not used computers before is helped over the initial stages. Also, some easy questions are set for the student to answer using the computer program.

Copies of the worksheets are included in the teaching guide.

The computer programs are written in BBC Basic and are suitable for the BBC model (B) microcomputer. They are written for colour monitor as well as for black and white. They required two 40 track discs. More details about the programming part of the materials are given in Appendix D.

Videotaped expositions

The use of video as a medium for learning has, in general, advantages and disadvantages. Video programmes can: be used as an open learning resource by adults or by students who are absent from the classroom because of illness; help students to understand some ideas with the use of language and of the visual image at the same time; be used for revision of previous knowledge; free the teacher for other tasks; provide experiences where first hand experiences are not possible. Besides the advantages which the use of video offers in the learning process

this medium also has some limitations. It presents a topic regardless of the reaction of the class. There is no interaction with the students, so they get bored quickly. The programme may not meet the needs of all the students, for some may be too slow while for others too fast. It takes a long time for preparation of the programmes and their production is usually costly.

In this study thirty-one videotaped expositions have been recorded. The way of recording and the presentation is very simple and does not aim at broadcast standards. It uses the 'voice-over' presentation described by Searl (1981). The teacher who records the programmes works step by step on some problems explaining as clearly as possible his/her work, using informal language. The teacher does not appear on the screen but only his/her written work appears on the screen. As reported by Searl, this method has been found successful for remediation and does not occupy much of the teacher's time.

The videotaped expositions differ from those described by Searl in some points. They cover the main parts of the content of the module $M_2|A_1$ and not just a selection of problems. In these programmes, the speaker appears sometimes on the screen, mostly when he/she describes an historical or introductory part of the lesson.

The programmes aim to cover the needs of the weaker students analysing as clearly as possible the mathematical concepts, structures and skills.

One of the aims of preparing these videotaped expositions of the course was to investigate whether the course content can indeed be covered within the teaching hours allotted to the module as described in the module descriptor and in the guidelines. It has been found

that the expositional part of the course covered nine hours and fourteen minutes, that is about one-third of the teaching hours allotted to the module, excluding the two last weeks of revision and remediation. The programmes can offer some ideas to the teachers about the distribution of the content of the module according to the time available. They can also give more detailed description of the content than is given in the module descriptor. They can also provide an example of a different teaching approach.

The expositions can be used for remediation by students working individually or in small groups. They can also be used as an open learning resource. Students who have missed a lesson can use the programmes. Another way of using them is by the class as a whole. This gives an opportunity to the teacher to modify the teacher-taught stance by sitting alongside the students watching the programmes, picking up points that might need further discussion and sometimes offering an alternative view. The programmes need to be used mixed with other teaching resources: by themselves they very quickly become boring. Back-up materials accompanying the expositions can provide practice of the main concepts and skills presented on the programmes. The programmes should not be used continuously for more than 10-15 minutes.

From the thirty-one expositions, nine have been recorded by the author while the rest have been recorded by Dr. John Searl, Senior Lecturer in the Department of Mathematics, University of Edinburgh. In Table 2.2, the number of programmes for each topic, their total duration and the number of teaching weeks for each topic are shown. More details about the content of the programmes and their presentation are included in the teaching guide (Appendix E).

Table 2.2

Topic	Number of programmes	Total duration	"weeks" (4h)
Errors	4	1 hr 15 mins	1
Functions and Graphs	2	34½ mins	2
Linear functions	5	1 hr 34 mins	
Quadratic functions	7	2 hr 26 mins	2
Trigonometry	9	2 hr 39 mins	2
Vectors	4	45 mins	1

The duration of each individual programme varies from 6½ minutes to 26½ minutes. The nine programmes on trigonometry are those recorded by the author. The commentary of those nine programmes is included in Appendix E.

Consolidation and Practice Exercises

A large number of students underachieve in mathematics. One of the causes of this underachievement is that they have not assimilated previous knowledge on which they can build new mathematical concepts and skills. Because of this they find the new material in their course very complicated and they have difficulty in coping with what they encounter in their class. Skemp (1971, 1986) states that for the formation of new concepts, contributory concepts needed for this abstraction must be available, "It is not sufficient for them to have been learned some time in the past, they must be accessible when needed". Cockcroft, also, in paragraph 248, emphasizes the need for practising skills and routines which the students have already learned so that they are available for use in other mathematical activities like problem solving and investigational work. Research also in Sudan (Sheikh, 1984), has shown that the regular practice of fundamental skills helped pupils to perform better in the examinations. Sheikh reported that about a quarter of the pupils from the failing group transferred to the passing group. In that study the consolidation exercises were a translation into Arabic of exercises taken from "Ten a Day" by A.L. Griffiths.

The importance of this recapitulative activity suggests that it is essential to provide the students in a formal way with suitable opportunities for the practice of skills, routines, concepts which they had met in their previous studies.

In this study a set of exercises modelled on the Griffiths pattern have been developed for the mathematics module $M_2|A_1$. Four booklets have been produced. Each

booklet consists of ten sets of ten easy questions. Answers, not solutions, are given at the back of the booklet.

The exercises are designed mainly for students who underachieve in mathematics. The aim is to improve the students' mathematical attainment by revising skills . . and ideas which they may have forgotten. They are partly diagnostic and can help both the student and teacher to identify areas of weakness. An attempt has been made to make the questions simple so as to give confidence to the students about their mathematical ability.

They cover basic materials taught in Secondary School mainly in the third year and earlier. Every set contains ten questions on a particular topic. A target time is given for each set hoping to induce a brisk attitude in the students towards the exercises. From the results of a pilot study, this target time seems to be appropriate for about 85% of the students. A typical set is shown in figure 2.1. In Table 2.3 the topics covered in each booklet are described. A more detailed description of the content of each booklet is given in the teaching guide (Appendix F).

The exercises are designed to be used by the students in their own time without any formal teaching. The students have to work through the booklets regularly doing one set of exercises per day. It is not advisable for them to work on more than one set of exercises per day because they quickly become a boring task. The order in which the set of exercises are to be used does not matter.

Figure 2.1

1. Simplify the expression $a^3 a^2$
2. Simplify the expression $a^4 \div a^2$
3. Simplify the expression $1 + x^{-5} x^5$
4. Simplify the expression $x^3 \div x^{-2}$
5. Simplify the expression $x^2 (x^3 + x^{-2})$
6. Simplify the expression $(x^2)^3 + x^2 x^4 + x^7 \div x$
7. Simplify the expression $(x + 1)(2x^3 + x)$
8. Simplify the expression $x^2 (x + 1) - x \cdot x^2 + 1$
9. Simplify the expression $(a + 1)^2 (a + 1)^{-1}$
10. Simplify the expression $((x + y)^3 \div (x + y)^2)(x + y)$

Target time: 9 minutes

Table 2.3

Booklet	Topic
1	Operations in numbers Properties of 2-dimensional shapes
2	Perimeter, area, volume of shapes Trigonometry
3	Simplification of algebraic expressions Solution of equations, systems
4	Functions, graphs Cartesian and polar coordinates, vectors

Practical activities

Practical work is a fundamental element in the learning of mathematics at all the stages of Education. "Mathematics is an abstract subject and becomes almost exclusively so too quickly for many pupils. Without sufficient practical experience the pupils are unable to refer abstract mathematical concepts to any form of reality. All pupils benefit from appropriate practical work of this kind whatever their age or ability", (Curriculum Matters 3,1985).

Practical work helps the students to understand concepts which are difficult to assimilate by other methods of learning, especially for the less able students. Bruner (1966) postulates that the building of a concept lies in three modes: enactive, iconic and symbolic. Each one plays a powerful role in the mental life of people at different ages. The interplay of these three modes constitutes one of the major features of adult intellectual life. The 'enactive' mode, the physical experience for building the concepts, is an important element for learning and understanding mathematical ideas.

Practical work involving the use of concrete materials, aiming to provide the physical experience needed by the pupils to understand mathematics, is often encountered in primary school classroom. Unfortunately it is usually neglected at the later stages of Education. This neglect of practical work has many causes, one of which is the tradition of presenting mathematics as a book-bound theoretical subject. The presentation of new ideas without any preparatory practical work causes students unnecessary difficulties in learning mathematics and they lose interest in the subject. Shayer and Wylam (1978) showed that the Piagetian stages of thinking do not generally follow the age levels Piaget identified.

A large number of students have not reached the stage of formal operations at the age of 16. They still need to manipulate concrete materials to understand concepts. Shayer and Adey (1981) also suggest that "in the adult population only 30% ever make use of theoretical models, or can handle multivariate problems, or can use any of the cognitive strategies characteristic of formal operational thinking". So, students who have not reached this stage find new ideas which are presented in a theoretical way meaningless. All students benefit from appropriate practical work not just the less able ones. This kind of work may create positive attitudes towards mathematics by showing applications to real life or by allowing students to discover by themselves important ideas and structures. Cockcroft in paragraph 247 says "Pupils of all levels of attainment can benefit from the opportunity for appropriate practical experience. The type of activity, the amount of time which is spent on it and the amount of repetition which is required will, of course, vary according to the needs and attainment of pupils".

Use of practical activities in the classroom can create an informal environment where the students can discuss and communicate mathematical ideas between themselves and with their teacher. This type of work also helps teachers to come closer to their students and to be able to diagnose their difficulties and help them.



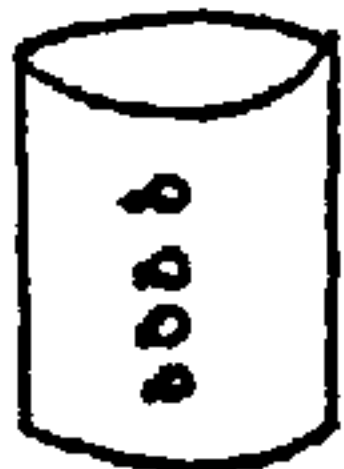
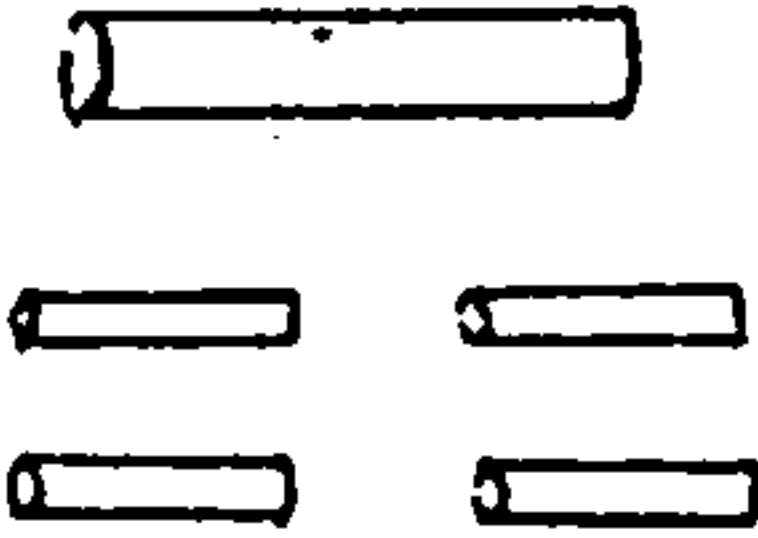
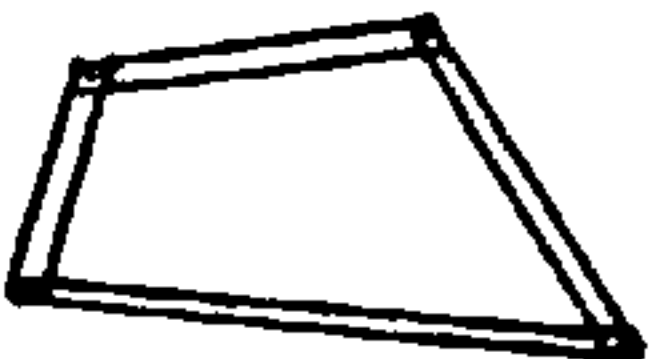
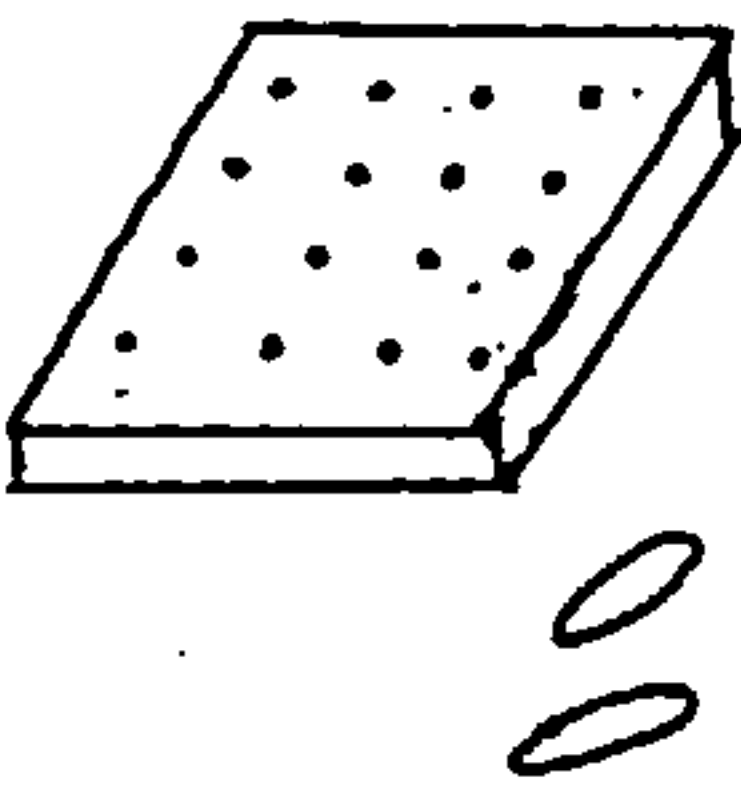
Sometimes the term 'practical work' has been misinterpreted. It has meant a collection of word problems about artificial situations without a real involvement of the student, for example, to collect some data. While some problem solving of this type may be necessary, it is no substitute for genuinely practical activities. In this study 'practical work'

has implied the use of concrete materials by the students to discover by themselves important concepts and structures of mathematics or appreciate the applications of mathematics to real life.

A possible danger of using this method of learning in the classroom is in making the activities over elaborate and so too time consuming for both pupil and teacher. This is particularly the case for the mathematics module $M_2|A_1$ where the duration of the course is quite short. The teacher also has to organise and direct the work of the students suitably so that the benefit from this activity is maximized. This type of work does not fit easily into a formal environment.

In this study twenty-four activities have been designed. In Table 2.4 a description and the teaching points of one practical activity for each topic of the module is shown. More information about the remaining activities is given in the teaching guide in Appendix G.

Table 2.4

Topic	Description	Teaching Points
Errors 	The 'fill depth' of a whisky bottle is investigated and the cost of each millimetre of fill above this level is obtained.	The importance of assessing the effects of errors in commerce and industry.
Linear function 	The time taken for a small ice block to melt is estimated and tested experimentally.	Fitting a straight line to experimental data.
Quadratic function 	The relationship between the rate of flow of water through a hole of a container and the distance of the hole from the bottom of the can is studied.	The investigation of a quadratic relationship from experimental data.
Graphs - modelling 	The number of small pipes which can be enclosed inside a protecting major pipe and the amount of material required to fill the unused space is investigated.	Drawing a graph from experimental data, area of circle, process of modelling.
Trigonometry 	The area enclosed by a jointed quadrilateral is investigated to find condition on angles for maximum.	Use of the area of a triangle $\frac{1}{2} ab \sin C$.
Vectors 	The representation of vectors in component form and properties of sum, difference, scalar multiple of them are investigated using a pinboard.	Sum, difference, scalar multiple of vectors represented by means of directed line segments and of components.

The main equipment of some activities was designed by the author and produced with the help of Mr. George Brown. The equipment is not very professional but is easy to use. The equipment for the rest of the activities can easily be found by teachers using the resources of their colleges or schools.

A description sheet and a record sheet accompany each activity. These can be seen in Appendix G. The structure of the description sheet is shown in figure 2.2.

Figure 2.2

16⁺ Active Maths

←

DESCRIPTION

INSTRUCTIONS

number of activity

drawing of equipment

Under the heading 'DESCRIPTION' the main aim of the activity and the equipment needed are described. Detailed instructions are given on the sheet about how to undertake the activity, together with some questions. The instructions and the questions are quite structured to help students carry out the activity easily without spending too much time. The record sheet is also structured containing tables or graph paper for the results of the activity.

A teaching guide has been provided for the teacher with the description of the activities, their teaching points, the equipment needed, comments on their use and copies of the completed record sheets. This guide is intended to help the teacher to identify the mathematical points of each activity and to organise and direct the students' work so that they appreciate the mathematics involved in each activity.

The activities are intended to be used in the classroom by the students themselves. It is preferable for the students to work in groups so that they share and communicate ideas. The outcomes of each activity have to be discussed so that the mathematical ideas may be assimilated and possible extensions be investigated. The activities can be used as a break between other formal teaching materials to offer a more relaxed atmosphere for the students. This is particularly the case with the timetable of the module in the colleges where the students cover four hours of the course in the same day.

An attempt has been made to produce a variety of activities and contexts so that the students are able to choose those which are closest to their interests.

Tape/booklet Sequences

These sequences consist of booklets with audio-tape commentaries which take the student through the mathematics in a step-by-step manner, each step requiring a response. They allow the students to work at their own pace through the booklet following the instructions on the tape and repeating steps which have not been understood as many times as they wish. They can also work through them in their own time without any help from the teacher. Their structure has characteristics of programmed learning methods like small step and quick response (Leith, 1964). The feedback which the student receives, working through the sequence, is the solution to a particular question in each step and is 'learner-paced'.

Giles (1981) supports the view that students may experience failure in mathematics not because of lack of ability but because of "lack of specific knowledge, techniques, ways of thinking and positive attitudes that are needed for success". Tape/booklet sequences can help the students to practice and assimilate techniques, skills and structures in mathematics and help them to perform better. They can also offer something in the area of positive attitudes. The students working by themselves through these individualised materials, and having the appropriate reinforcement in each step gain confidence in tackling similar problems in mathematics. This method of learning has been used successfully by university students as described by Searl (1975).

The tape/booklet sequences can be mainly used by students who have difficulty in mathematics and usually for remediation. Sometimes all the students can benefit by revising certain skills.

Twelve tape/booklet sequences have been produced in this study, based on the lines that Searl (1975) suggested. These sequences cover important mathematical skills and structures of the module $M_2|A_1$. A summary of the content of each sequence is shown in Table 2.5.

Table 2.5

Title	Content
Rounding errors	Rounding off, error bounds, errors in arithmetic operations, correctly rounded number.
Graph of linear function	Graph of $y = mx + c$, graphical solution of linear equation and system, gradient, y-intercept.
Finding the formula of a straight line	Finding the formula, if one point of the line and its gradient are known or two points of the line are known.
The graph of the quadratic function	Evaluation of quadratic functions, shape of graph, turning points, axis of symmetry, y-intercept, graphical solution of $ax^2 + bx + c = 0$.
Solution of quadratic equations by factorization	Equations of the form $(ax + b)(cx + d) = 0$, $a, b, c, d \in \mathbb{R}$, solution of $x^2 + bx + c = 0$, $b, c \in \mathbb{R}$ using that the product of the roots is c and their sum $-b$.
Solution of quadratic equations by completing the square and by formula	Solution of $x^2 = d$, $d \in \mathbb{R}$, solution of $ax^2 + bx + c = 0$, $a, b, c \in \mathbb{R}$ by completing the square and by formula (existence of roots by discriminant).
Sketching of the graph of a quadratic function	Shape of the graph $y = ax^2 + bx + c$, $a, b, c \in \mathbb{R}$, points where the graph cuts the axes, turning point, equation of axis of symmetry, sketching of graph.

CONTD./

Table 2.5 (contd.)

Title	Content
The trigonometric functions on your calculator	Use of calculator to evaluate trigonometric expressions for angles in degrees or radians, calculation of angle knowing its trigonometric value.
Trigonometry and triangles	Solution of right-angled triangle, sine rule, cosine rule, solution of a general triangle.
Graphs of the circular functions	Definitions and graphs of $\sin \theta$, $\cos \theta$, $\tan \theta$, properties of graphs, graphs of $a \sin \theta$, $a \cos \theta$, $\sin(n\theta)$, $\cos(n\theta)$, $\cos(\theta + \phi)$.
Circular measure	Relationship between arclength and subtended angle, definition of radian, change of angles from degrees to radians and vice versa, applications.
Vectors	Vectors in two dimensions expressed as directed line segments and in component form, operations, magnitude.

The content of each sequence is decided by taking into account points of the course where the students face a particular difficulty. After the content has been decided, the examples which the students work through are chosen. In figure 2.3 two pages from the booklet on Rounding Errors and the commentary for these which is recorded on the tape are shown. In one page (49) an example for evaluating an expression involving rounded numbers is shown. After an introduction to the method and describing how the student can work it out, the student is asked in question 25 to continue working on this particular example. He/she starts working on this question after hearing the sound "BUZZ" on the tape. He/she then has to stop the tape, work through the question and check his/her answer on the next page of the booklet (page 50). Finally, he/she starts the tape and continues on to the next step. In the next step the student will be asked to carry on to the next stage of this particular example. Later, in another part after practising these steps, he/she will face an example which integrates all these steps together. With this integrative example which contains skills which he/she has practised individually, he/she can revise the skills in a holistic way. Sometimes it is possible to adopt the method of having an example on the page and asking the student to do a similar example. This method has to be used with care because it can become boring. It is preferable to lead the students through the process which they need to follow for the particular question. After each question the students are reminded in the booklet not to turn over the page until they are ready to check their answer. On the cover of the booklet instructions are given about how to use it. In the commentary, at the beginning, instructions about what equipment the students need and what the booklet is about are given.

Figure 2.3

Rounding Errors

Booklet

Commentary

Page 49

$8.20 - 2.51 \times 1.01$

The diagram illustrates the order of operations for the expression $8.20 - 2.51 \times 1.01$. It shows three initial numbers: 8.20, 2.51, and 1.01. The numbers 2.51 and 1.01 are first multiplied (indicated by a bracket labeled 'x' and the result 2.5351). This product is then subtracted from 8.20 (indicated by a bracket labeled 'd' and the result 5.6649). The final result is labeled 'd-c'. Brackets on the left side of the diagram are labeled 'order' and 'label'.

Draw a table with columns the labels, numbers, absolute error bounds, relative errors.

label	number	absolute error b	relative error b
a			
b			
a x b = c			
d			
d - c			

On page 49 you see the expression 8.20 minus 2.51 times 1.01 where 8.20, 2.51 and 1.01 are correctly rounded numbers. We want to evaluate this expression. We do this in the following way:

Firstly we decide the order in which the calculation will be done. It is good to draw a diagram to show the order. For example, we write the three numbers 8.20, 2.51 and 1.01 one below the other. The two numbers 2.51 and 1.01 are multiplied first and we get the answer 2.5351 if they were exact. The second operation is the subtraction of the product 2.51 times 1.01 from the number 8.20. This gives us the result 5.6649. After deciding the order of the operations we label the numbers to clarify the work. We put 'a' as the number 2.51, 'b' as the number 1.01. The result of the first operation 'a' times 'b' we call c. We label the number 8.20 'd' and so the result of the subtraction is the difference of 'd' minus 'c'.

CONTD. /

Figure 2.3 (contd.)

Question 25

Copy on your paper
the table and
calculate the
absolute error
bound of the
product $a \times b$
writing your steps
on the table.

DO NOT TURN OVER
UNTIL YOU ARE READY
TO CHECK YOUR ANSWER

Thirdly we draw up a table on
Page 49 and we put the labels
and the numbers in the order
which we have previously decided
upon. In the questions 25 and
26 you will be asked to complete
the table in order to find the
absolute error bound of the
expression and to write it in
terms of its extreme values.
Remember that the absolute
error bound of the sum and of
the difference of two numbers
is calculated by adding the
absolute error bounds of the
numbers. The relative error
bound of the product or quotient
of two numbers is calculated by
adding their relative errors.
We get the absolute error by
multiplying the relative error
by the value of the product or
quotient. Now try question 25.
BUZZ.

Page 50

Answer 25

CONTD./

Figure 2.3 (contd.)

Answer 25

label	number	absolute error b	relative error b
a	2.51	0.005	0.002
b	1.01	0.005	0.005
axb=c	2.5351	<div><div>0.02</div><div>x2.5351</div></div>	0.007
d	8.20		
d-c	5.6649		

In the commentary the language is informal but carefully used so as not to create misconceptions. When the booklet has been constructed the commentary is written and recorded. Then the sequence is tried out by someone familiar with the content and after some changes, the final version is produced.

These materials can be used for remediation or revision. They can be used by the students in their own time at home or in the library of the school or college or in the language laboratory. They can also be used in the classroom. It is preferable and sometimes necessary for the students to wear headphones so that they do not distract others and they are not easily distracted themselves.

A teaching guide has been produced giving information about the content of the booklets and their possible use. It is included in Appendix H.

The booklets and the tapes are included in Appendix H.

Posters.

Posters can be used effectively as a resource for learning mathematics. In the classroom the use of posters can create a suitable atmosphere which provides motivation and interest to the students. "The atmosphere in which mathematics is taught ought to be conducive to learning, with stimulating and attractive display of the above resources together with the display of pupils' work and of objects and posters of interest." (Curriculum Matters 3, 1985).

Sheikh (1984) describes a successful use of a set of very simple posters in primary schools' classrooms. As he concluded from the comments of the teachers, the use of posters as a teaching medium motivated the children and reinforced their learning.

Posters can act as stimulators of mathematical activities. They can create real discussion between pupils themselves and between teacher and students which can offer rich mathematical experience. Jaworski (1985) describes a use of posters which created discussion and helped her thirteen year old pupils to work cooperatively.

Another use of posters can be seen at university level (Searl, 1985). He describes some selections of posters for exhibition outside the lecture theatres which generated interest among the students.

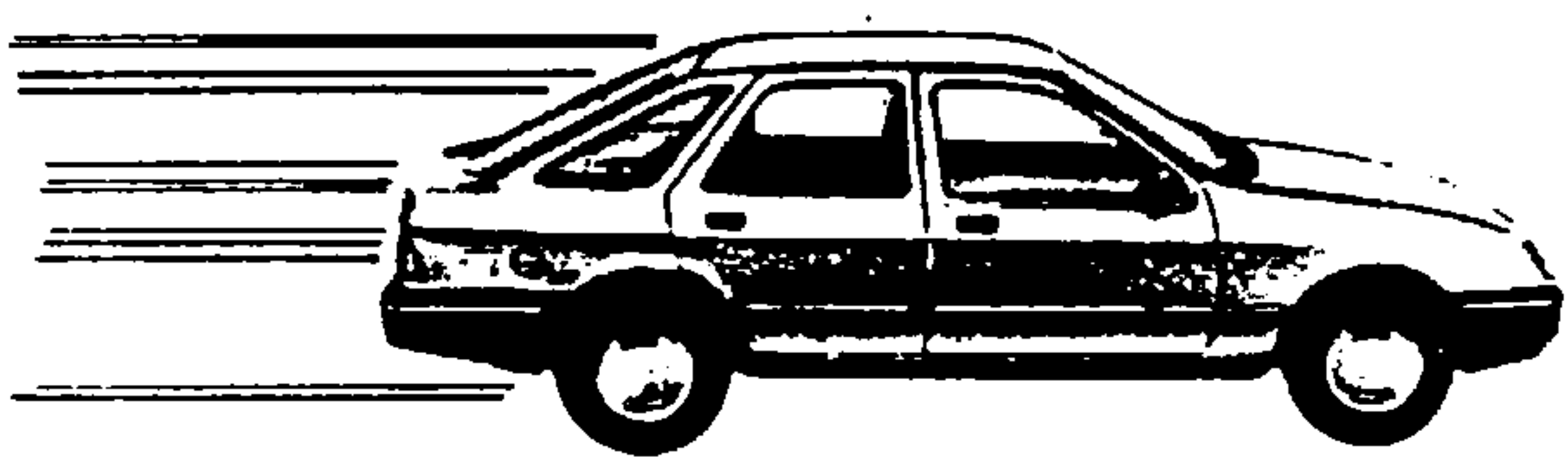
Generally, posters can broaden the student's mathematical experience. They can provide the students with background information which would otherwise be inaccessible to them either because the source of information is too diverse or because of the way text books intimidate many students. By providing this information in an attractive way, students will unconsciously do the background reading. They can also encourage good study skills illustrating for example how to set out a calculation or document a computer program etc. The presentation of puzzles or open problems can give opportunity for an investigation. Students can also appreciate applications of mathematics or revise facts which can easily be forgotten. Suitable use of visual images on the posters can help students to form mathematical concepts.

In this study a series of eight posters have been designed for the module $M_2|A_1$. The topics covered are relevant to the content of the module. A first draft of the posters had been produced by the author and the final version of the posters were produced with the help of the Audio-Visual Services of Edinburgh University. The posters use two colours for attractiveness and clarification of some images. The A3 size of the posters was

chosen so that the cost would be reasonable and they could be easily used in the classroom.

A description of each poster follows.

===== ERRORS =====



===== 'One hundred metres in four seconds' =====

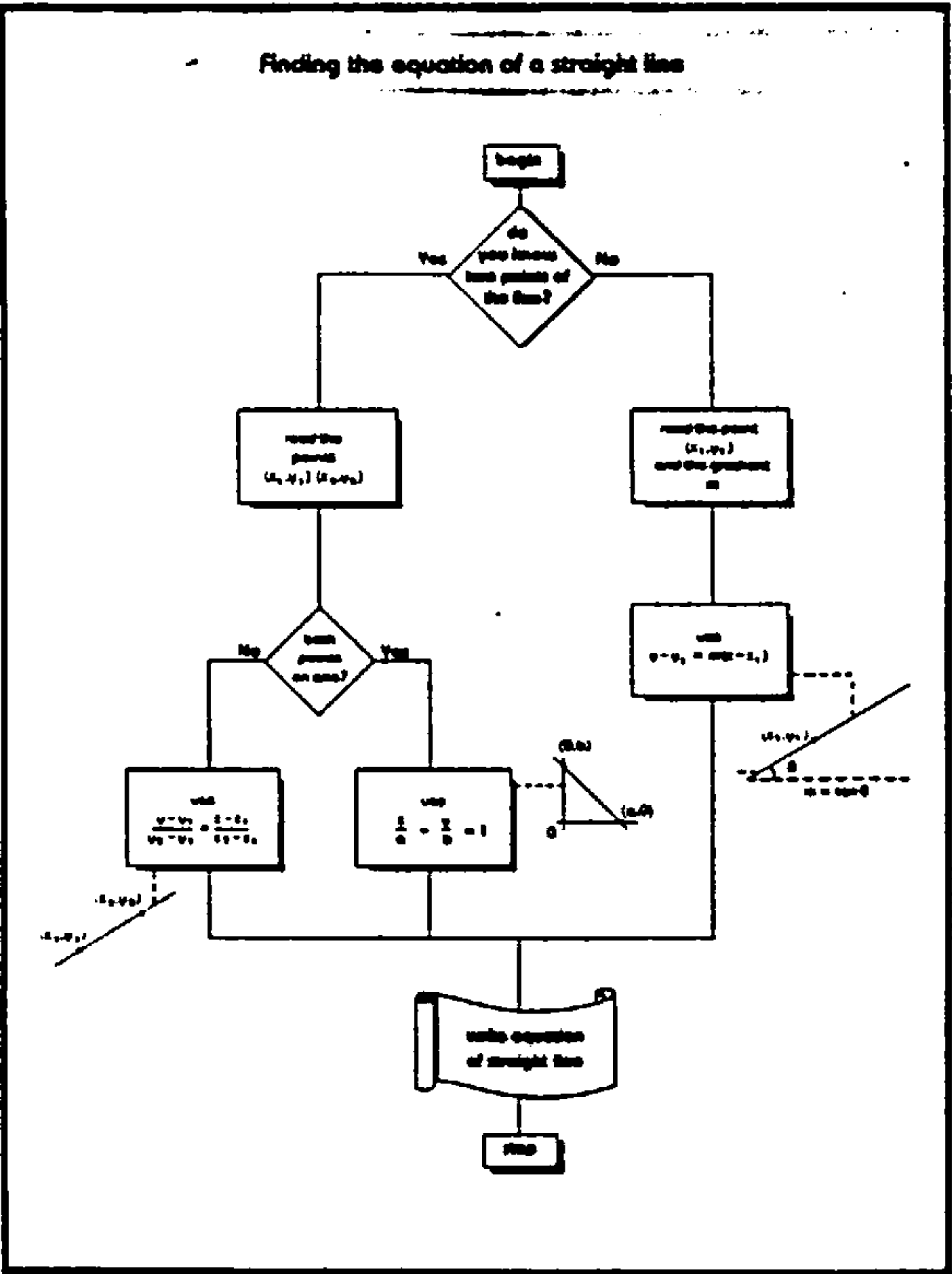
What is the speed?

90km/h?

Accuracy		DISTANCE		
		nearest metre D = 100 m	nearest centimetre D = 100.00 m	nearest millimetre D = 100.000 m
TIME	Nearest Second T = 4 sec	Speed = 90 ± 11.7 km/h	Speed = 90 ± 11.3 km/h	Speed = 90 ± 11.25 km/h
	Nearest tenth of a second T = 4.0 sec	Speed = 90 ± 1.58 km/h	Speed = 90 ± 1.13 km/h	Speed = 90 ± 1.25 km/h
	Nearest hundredth of a second T = 4.00 sec	Speed = 90 ± 0.56 km/h	Speed = 90 ± 0.18 km/h	Speed = 90 ± 0.11 km/h

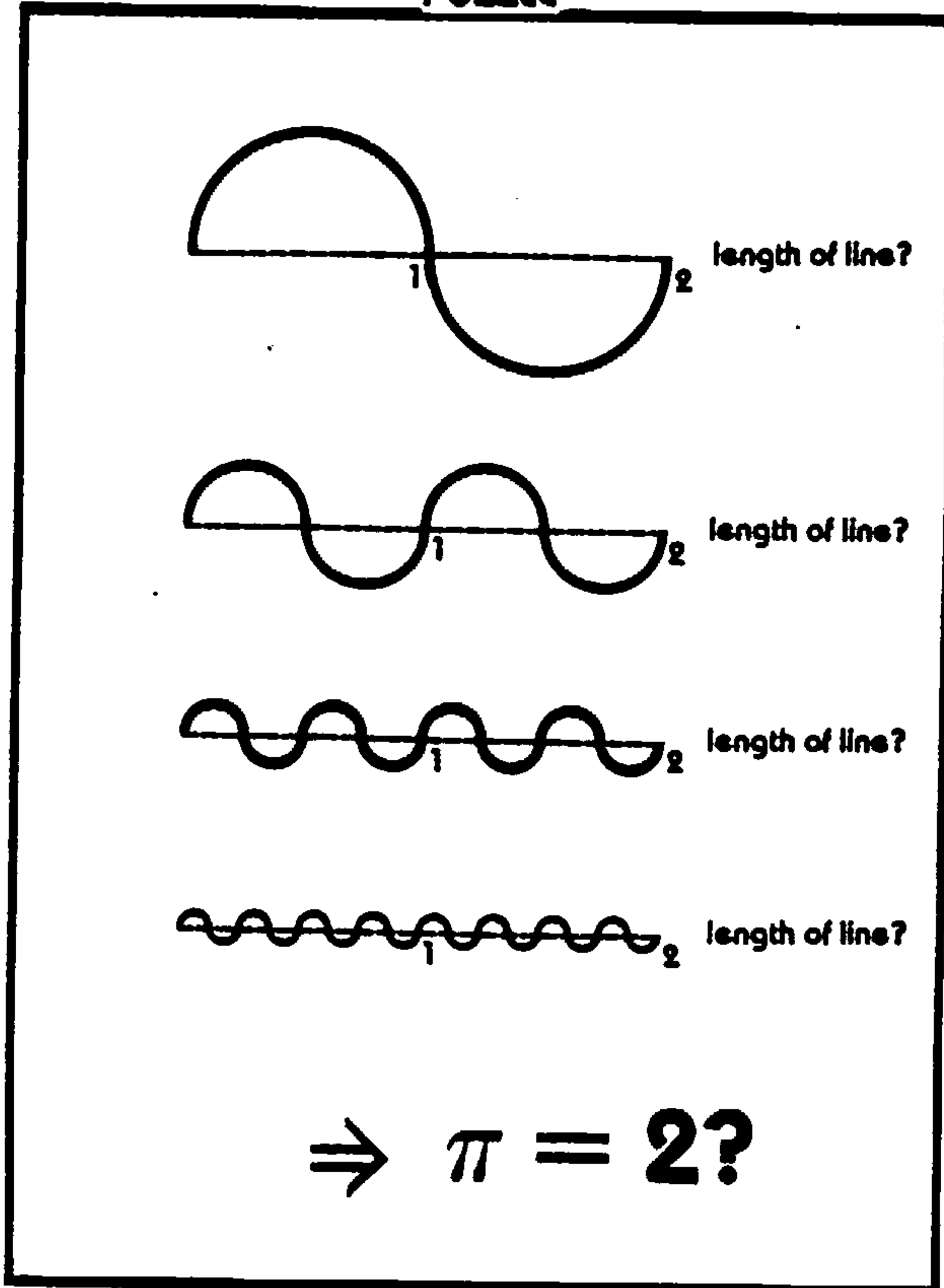
Edinburgh Mathematics Teaching Group
University of Edinburgh

The poster 'Errors' attempts to help students appreciate the importance of the accuracy in practical problems. It shows that more accurate measurements give results with smaller error. It also emphasizes that an answer (90km/h) which the arithmetic of exact numbers gives is not the answer to the real problem.



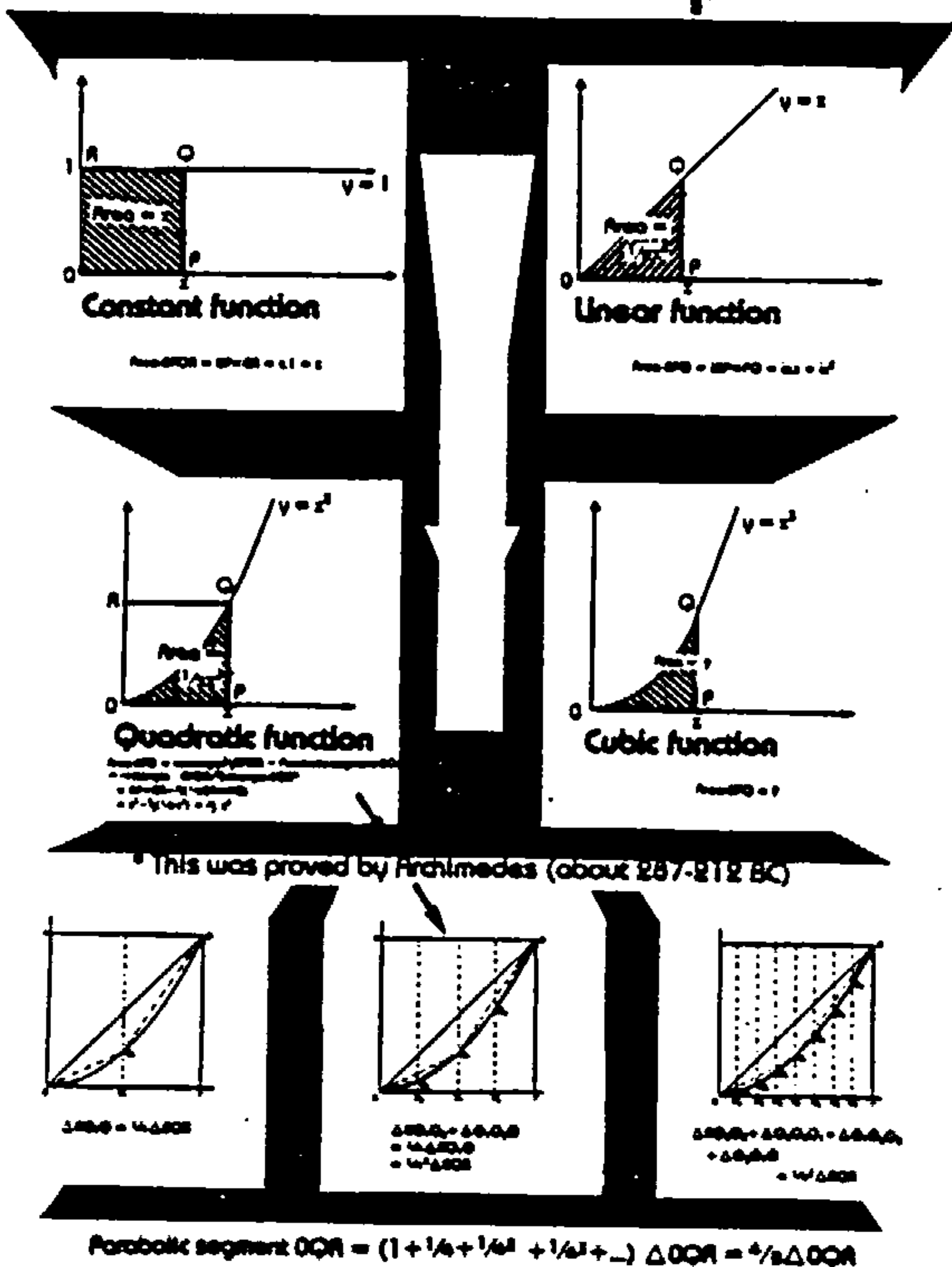
The poster 'Finding the equation of a straight line' can help students to revise methods which they easily forget. It shows these methods in a systematic way using a flowchart.

PUZZLE



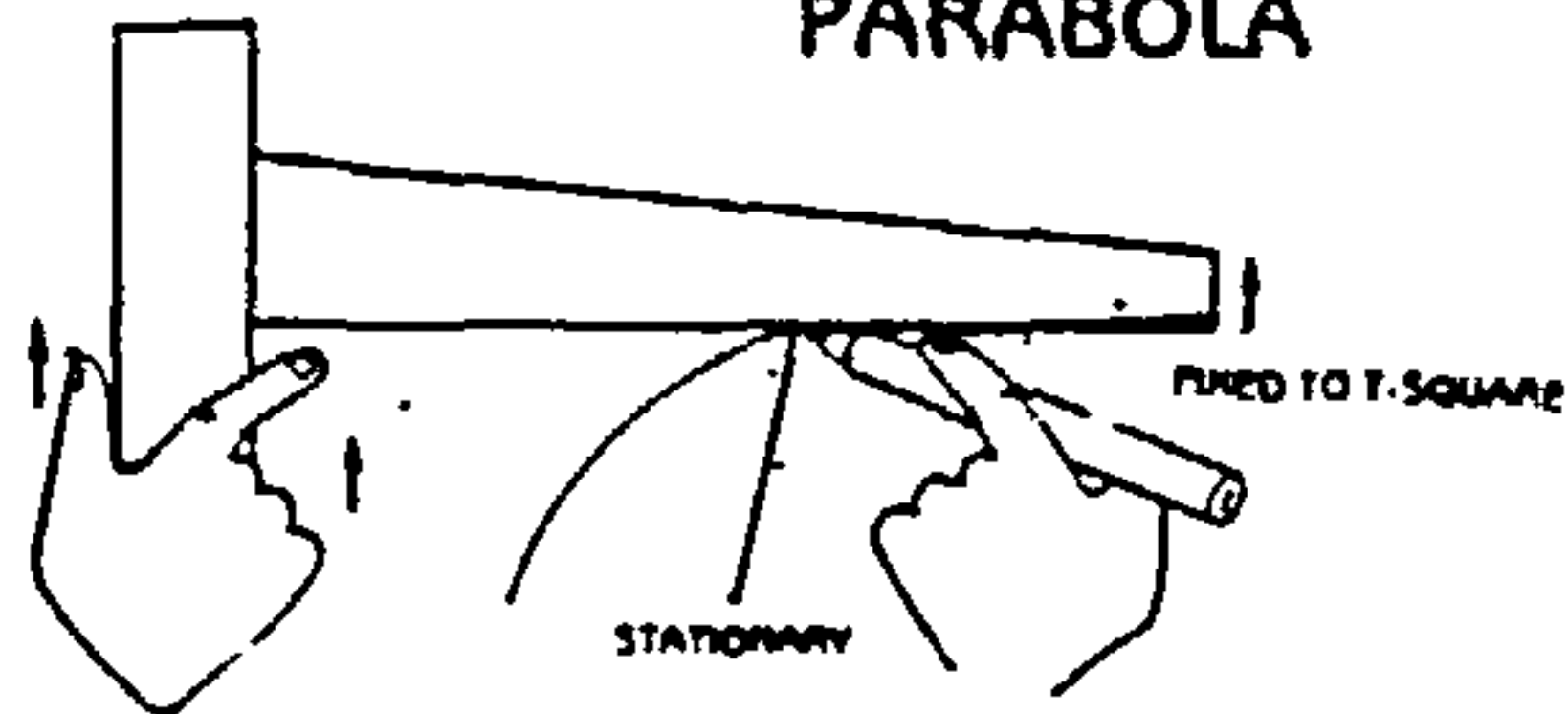
The poster 'Puzzle' can stimulate discussion with the extraordinary result that $\pi = 2$. In this stage the students cannot give a proof but they can give some explanations with the teacher's help. Some mathematical skills are also practised. An extension could be the calculation of the area between the bold and dotted line.

Area under Graphs



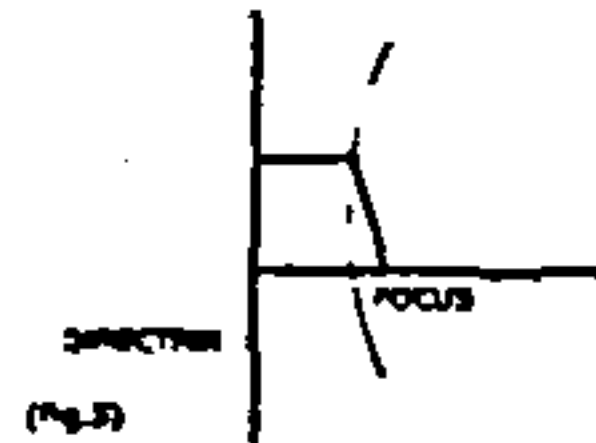
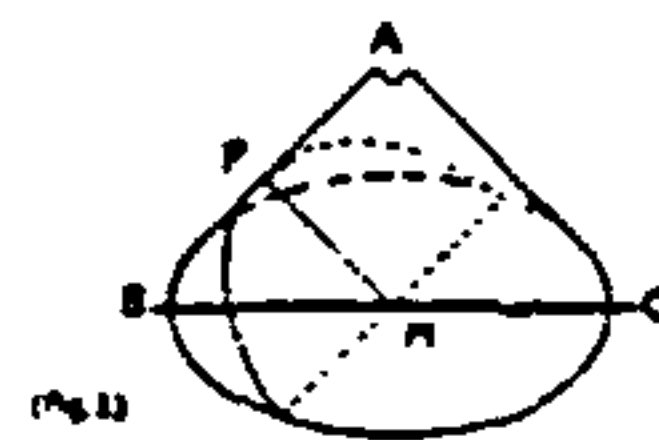
The poster 'Area under Graphs' prepares the students for calculus concepts which they will meet later in their studies. It approaches the problem of area under some curves using geometric methods. An investigation is set out for calculating the area under $y = x^3$. It also provides students with background information from the history of mathematics.

PARABOLA



The parabola was studied by the Greeks as one of the sections of a cone. Menaechmus (c.350BC) formed a parabola by cutting a right-angled right cone by a plane at right angle to one of its sides. (fig.1)

Apollonius (c 220BC) produced parabolas by cutting any cone by a plane perpendicular to a diameter of the base of the cone. (Fig 2)



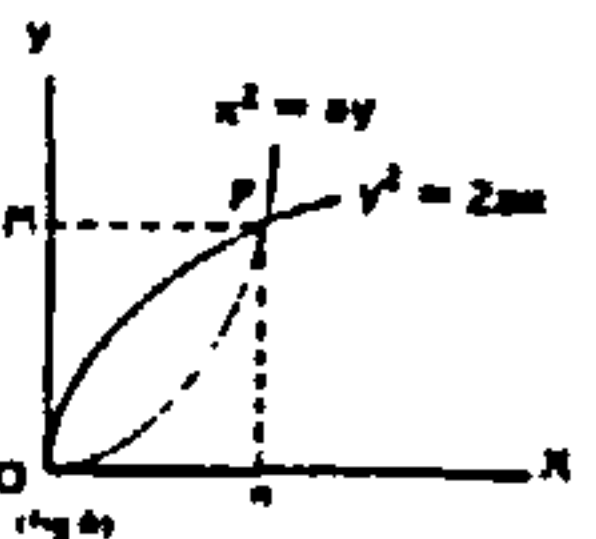
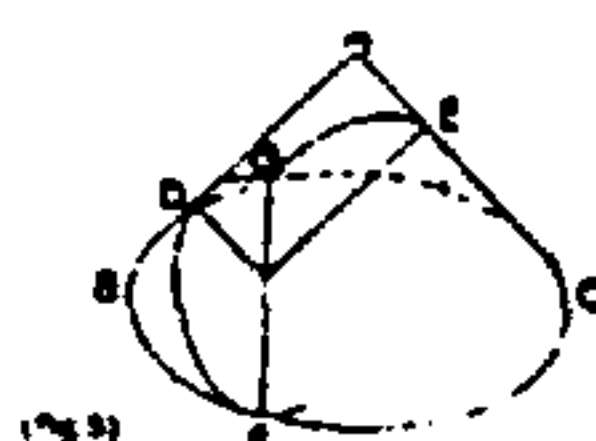
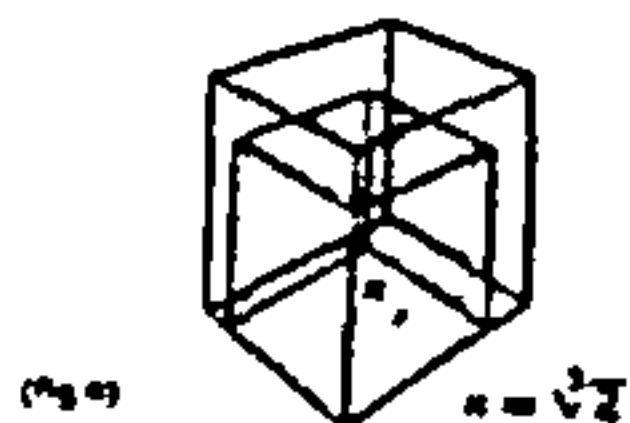
Pappus (540AD) defined the parabola as the set of points whose distance from a fixed point (the focus) and from a fixed line (the directrix) are equal (fig.9)

THE DUPLICATION OF THE CUBE

This problem may have originated in an ancient Greek poem in which the mythical King Minos is dissatisfied with the size of tomb erected to his son Glaucus. He ordered that the tomb be doubled in size. (Fig 4) Minos stated incorrectly, that this could be done by doubling each dimension of the tomb. This faulty mathematics led the geometers to take up the problem.

Minpocrates reduced the problem to the construction of two mean proportionals between two given line segments of lengths a and $2a$

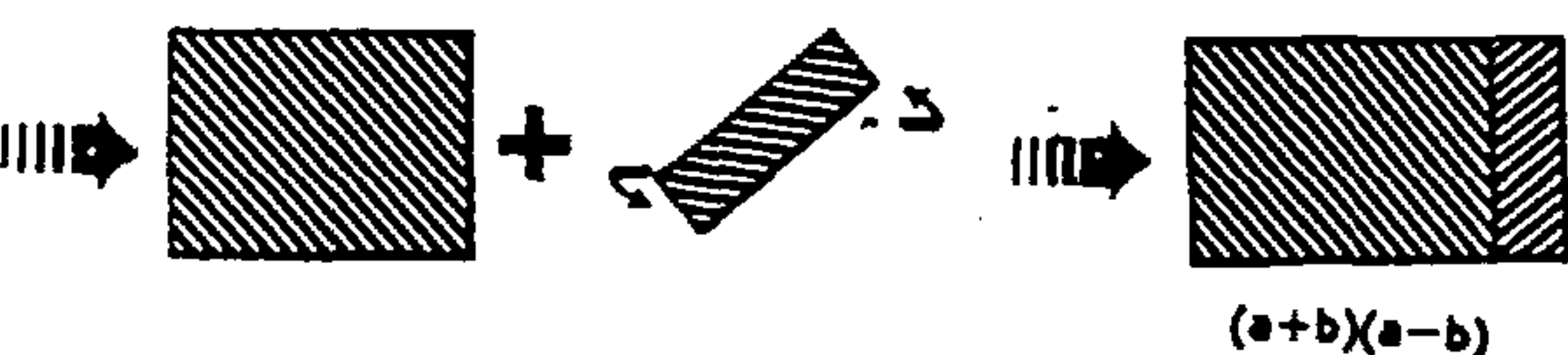
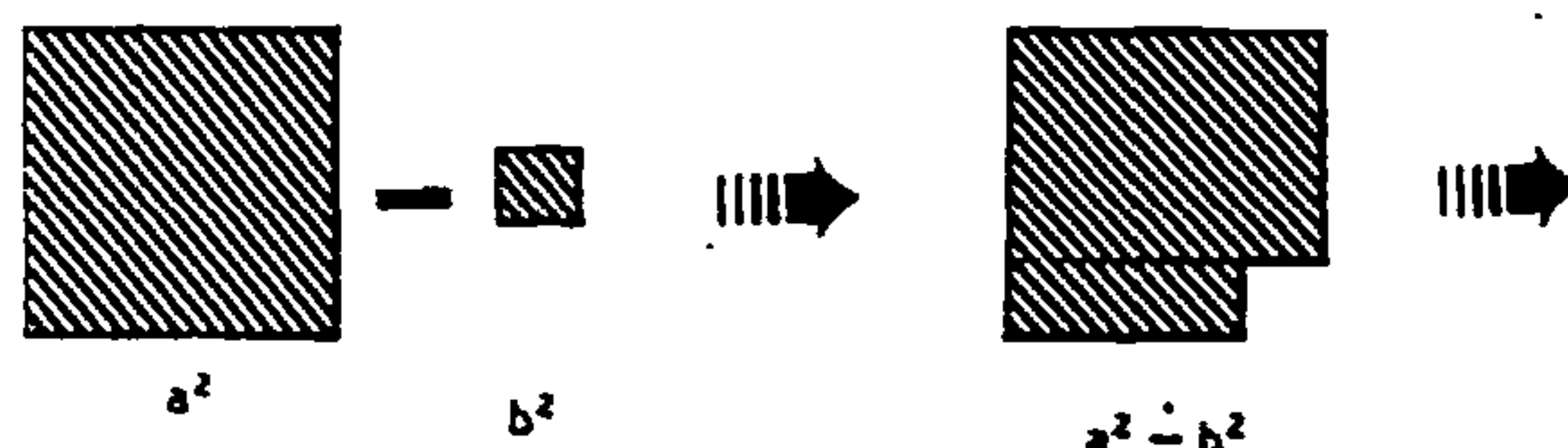
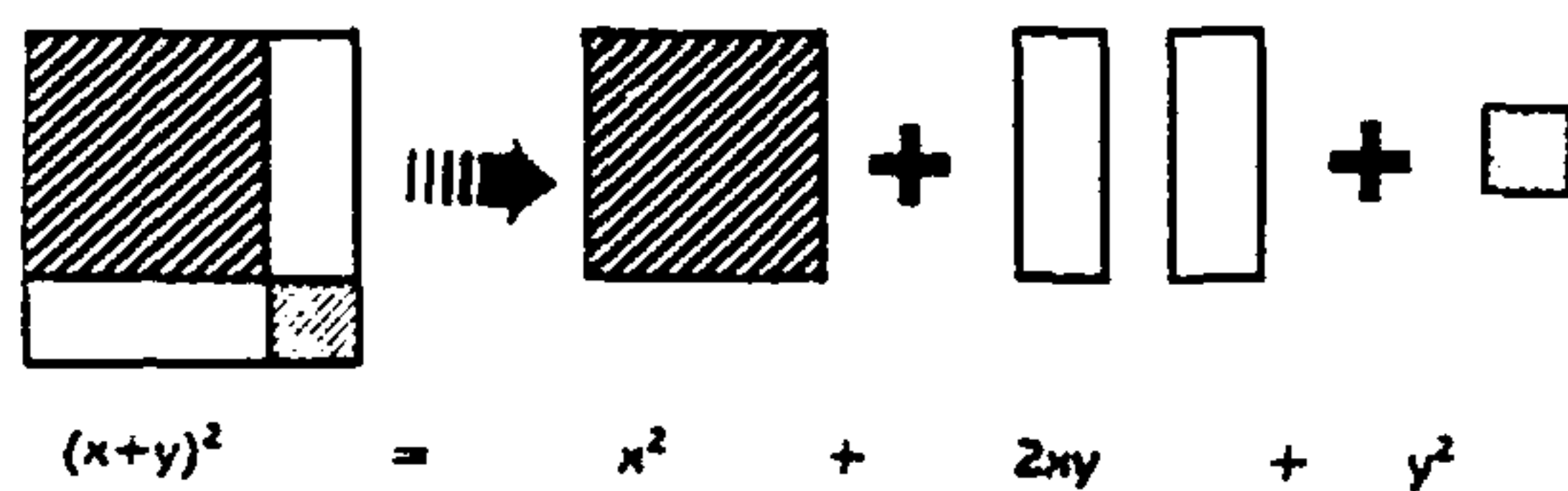
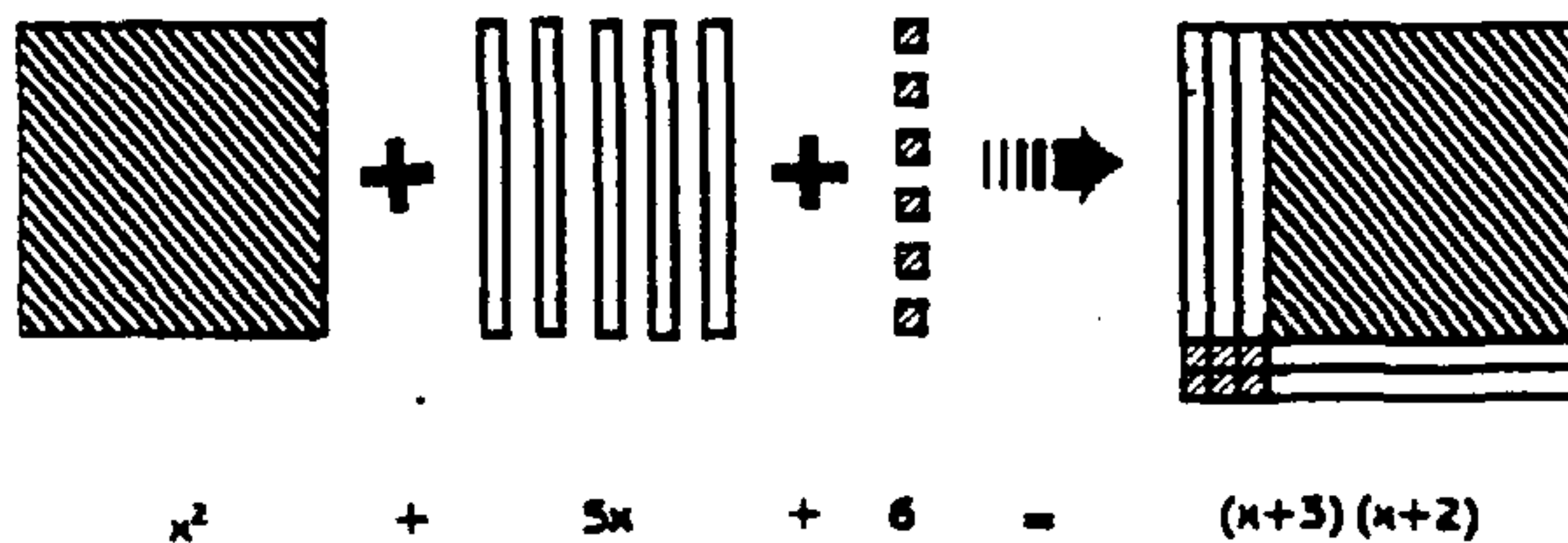
that is to find the lengths x, y such as $\frac{x}{\pi} = \frac{\pi}{y} = \frac{y}{2a}$



The intersection P of the two parabolas defines the segments ON , OM which are the required means μ, ν . (fig.6)

4-10-1964

Factorising Quadratic Expressions



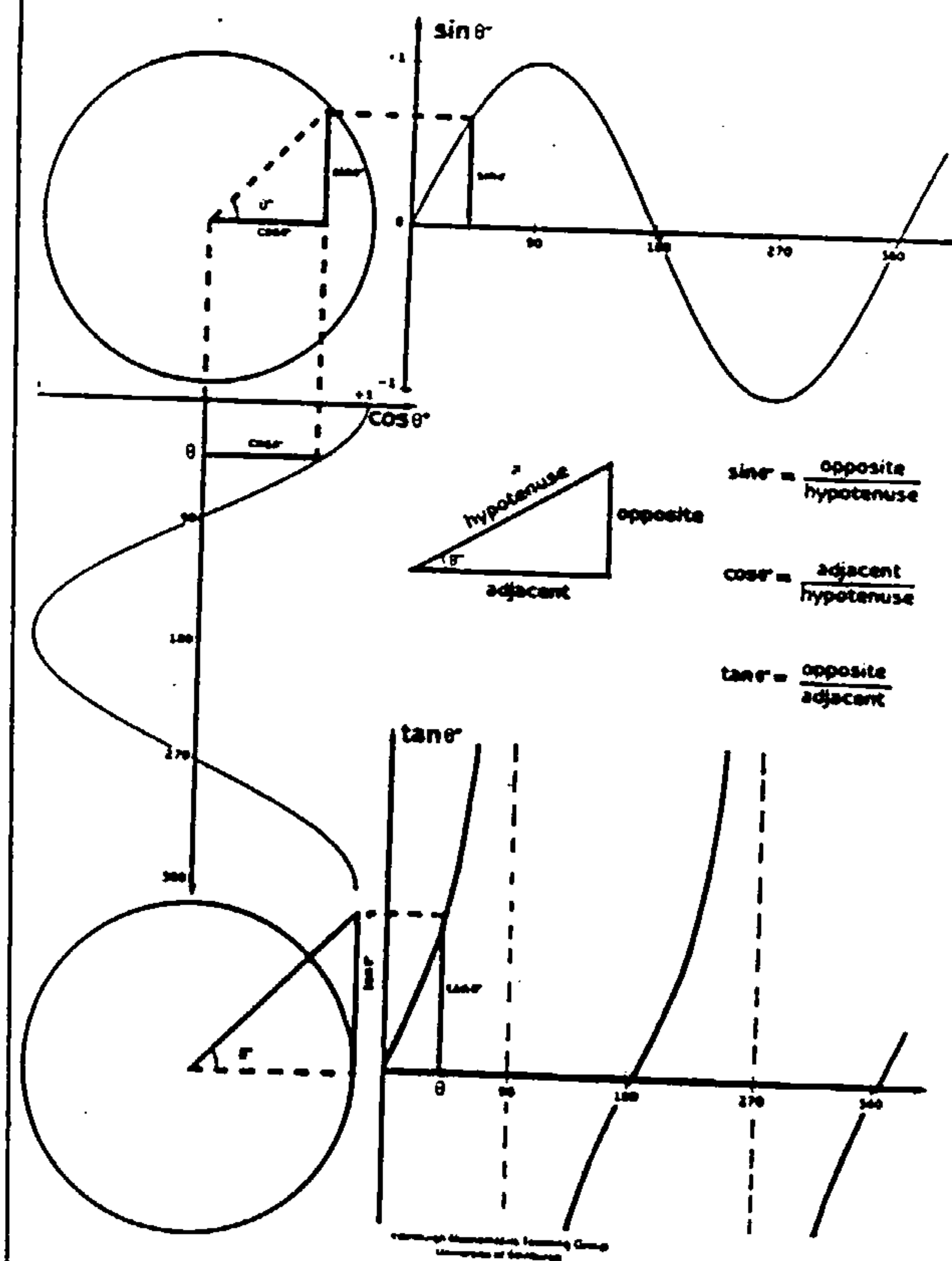
The poster 'Parabola' gives some historical background about the parabola. It may help students to appreciate the process of creation of mathematics. Some geometric properties of the parabola are emphasized.

The poster 'Factorising quadratic expressions' gives a geometrical presentation of the factorization. The expressions

$$(x + y)^2 = x^2 + 2xy + y^2 \text{ and}$$
$$a^2 - b^2 = (a + b)(a - b)$$

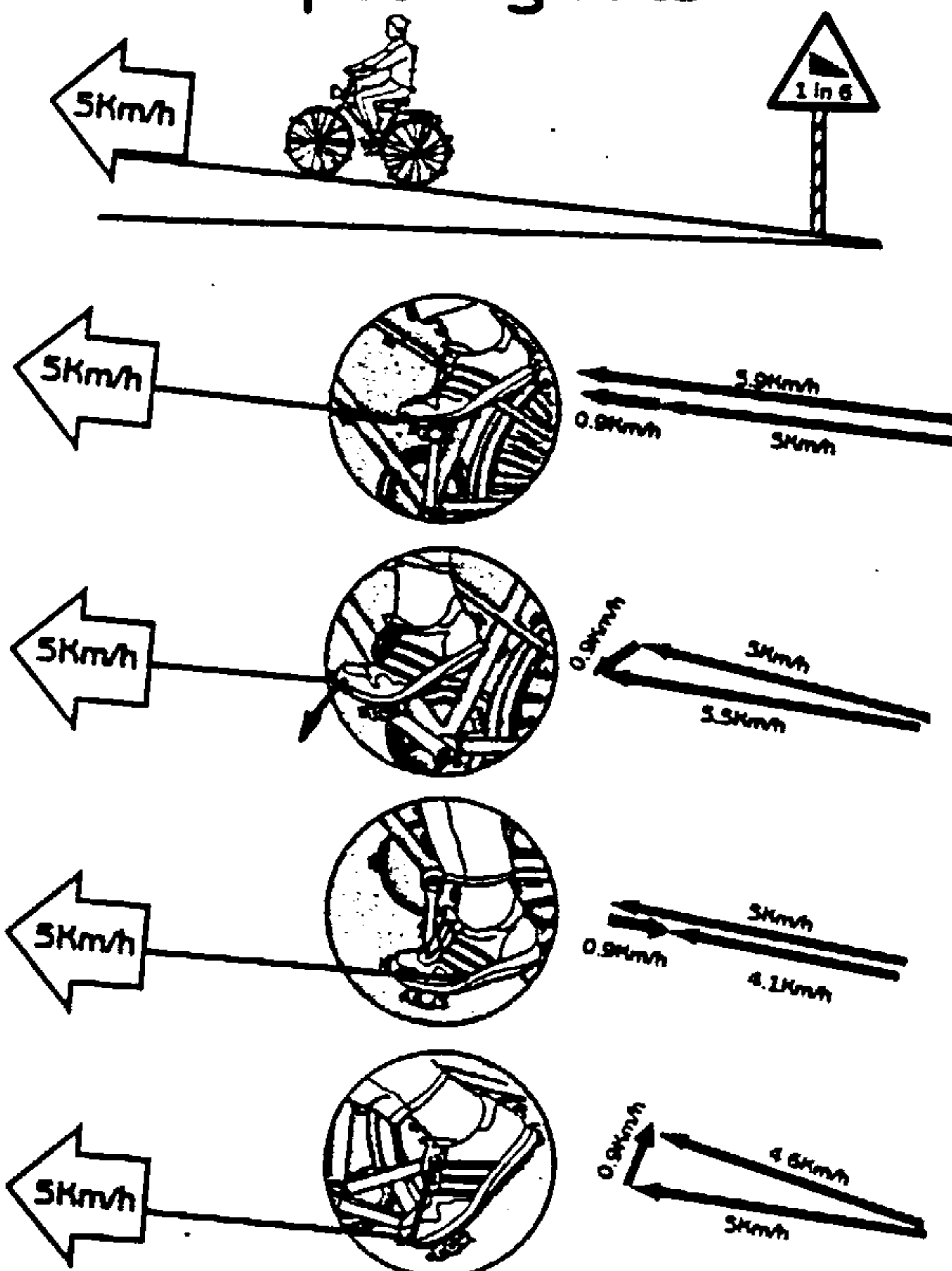
can be used for factorising an expression by completing the square.

The Circular Functions



The poster 'The Circular Functions' shows two ways of defining the trigonometric ratios, one in terms of the rotating radius in a unit circle, and the other in a right-angled triangle.

Speeding Toes



The poster 'Speeding Toes' shows an application of vectors in a real problem. It uses a problem usually familiar to most students, so it can create interest. The problem is about calculating the speed of a cyclist's toe.

As can be seen from the description, an attempt has been made to produce a variety of ideas and ways of presentation. Some posters show practical applications of mathematics while others present concepts without using a practical context. Some leave open questions for discussion and investigation, while others provide background information in a descriptive way. The explanation of algebraic structures using geometrical methods is used in two of them.

A teaching guide has been produced which gives some suggestions for use of the posters in the classroom and provides a short description of each poster. The teaching guide and the posters are included in Appendix I.

The teacher must place the posters where they can be easily seen. The students do not always respond to the posters by themselves, so that the lecturer's intervention is sometimes necessary.

Investigations

Investigational work is a fundamental activity for the learning of mathematics. It can provide an opportunity for the students to integrate newly acquired knowledge and skills within their prior knowledge and experience. Cockcroft in paragraph 250 says "The idea of investigation is fundamental both to the study of mathematics itself and also to an understanding of the ways in which Mathematics can be used to extend knowledge and to solve problems in very many fields".

The meaning of 'investigation' has been interpreted differently by different authors. In some cases it has also been misunderstood. The difference between investigational work and problem solving activity is not easily clarified. Wells (1985)

argues strongly about the differentiation between mathematical problems and investigations. In the author's view an investigation is a problem solving activity but usually of a more integrative nature. It integrates different skills, concepts, structures in different contexts. It can be open-ended or more structured and it emphasizes mainly the mathematical process itself. 'Investigation' also can be something more general, a search for information for example to find the history of $\sqrt{2}$, or an art work for example to investigate different methods for constructing a parabola.

Investigations do not always have 'one solution' but may provide a search for the best solution to the problem. Burton (1980) states "More usually problems are open-search in the sense that the method chosen to deal with them is best-fit rather than exact and relies upon an amalgam of objective and subjective information or even chance. What is then obtained is not a "solution" but a personal resolution of the problem which the individual judges not as right or wrong but as adequate." This point makes clear that investigational work is not an activity only for the most able students but for all the students. Everybody can work through it and make progress although the level of mathematics used or the solution will differ from student to student.

One of the main aims of the investigational work is to produce independent learners. The students have freedom to work at their own pace and choose the method which they regard as most appropriate to tackle the particular problem. They are not forced to work in a particular way which is usually the case with formal methods of teaching and with routine problems.

What are the processes of mathematics which a student has to appreciate and practice, is a very general question. Scottish Examination Board (1984) states that interpreting information in a problem, selecting a strategy to tackle it, processing data and communicating the results are the objectives for a problem solving activity in the Standard Grade Course. In addition to these, Burton (1980) regards as important for solving a problem the ability to collect, classify, analyse and use information, search for relationships, make and test hypotheses, discriminate between objective and subjective information, specialise, work systematically, verify the results, derive a statement about the perceived pattern of results. Ganderton (1985) regards that the processes of mathematics include recognition of the problem, systematic breakdown of the problem, pattern searching, tabulating, graphing, recognition of patterns, conjecturing, symbolising, generalising, hypothesis testing, justification and proof, application and communication. Ability to estimate and approximate are also important elements of the mathematical processes. Suitable choice of investigations can help students to practise these processes of mathematics. An investigation must give opportunity for practising most of the processes if it is to be regarded as effective for learning. Of course, it is not possible for one investigation to cover every aspect of the mathematical process. It is the variety of different investigations which helps students to appreciate mathematics.

One of the most important characteristics of the mathematical investigation is the communication of the ideas, methods and results of the investigation. This can be in the form of discussion or in written form. Both are essential elements for the mathematical development of the students and for their future

needs in the society.

The discussion can take place in the classroom and the results, methods of tackling the investigations and possible problems can be discussed between the students themselves and between the students and the teacher. From a creative discussion everybody can gain a mathematical insight clarifying and extending his or her ideas.

The students can also benefit from the writing of a report about the investigation which they have tackled. This provides opportunity for the teacher to assess more easily the students' work, to give them suitable feedback so that they will perform better in another piece of work. The writing of the report also helps students to clarify their ideas and appreciate the importance of presenting their work in an intelligible way. It also provides 'good practice' habits like presenting their results in tabular form, drawing clearly graphs, giving references to sources of information, etc.

The investigative approach may also provide positive attitudes towards mathematics. It may develop the students' mathematical self-confidence so that they are prepared to tackle non-routine problems. It may also provide motivation. Of course this depends on the type of investigation and its relevance to the students' interests.

Students need encouragement and support from their teacher with this unfamiliar work. Because their experience with mathematical problems is mainly to look for 'the answer', they do not feel 'safe' with this new method of learning. Suitable direction is necessary by the teacher especially at the first stages of the work. Evans (1987), states that "In the early

days of investigative work there should be plenty of guidance of all pupils, but with a gradual "weaning off process." Wells (1985), says "a premature emphasis on the solver's autonomy frequently produces stress and anxiety in the solver". This does not give a reason for not using investigations but shows the importance of the teacher's role in this activity.

Although the use of investigational work is very important in the learning of mathematics, it is often neglected as a teaching approach. The main reasons for this are that teachers do not have experience in this method of working and they feel insecure when they face problems which do not have clearly a deductive approach. In addition to this, the assessment of the students' work is quite difficult. It occupies a lot of the teacher's time and the criteria are not 'correct or wrong answer' but more vague. Students tackling the problem with quite different methods and mathematical concepts and skills can produce the same good pieces of work. The process of giving an appropriate feedback either in written or in oral form requires a good mathematical background and careful consideration in order to react positively to the students' performance. The students have also difficulty in appreciating the usefulness of this type of work. They often say "It is not proper maths".

Despite the difficulties in using investigations in the classroom, their role in the mathematical development of the students cannot be neglected. Burghes (1984) states "Constructive uses of investigations in the classroom should improve pupils' problem solving strategy and ability - and these characteristics will be of help in the future, when an ability to cope with a rapidly changing technical world and its associated problems will be needed."

The use of investigations is suggested both in the module descriptor and in the guidelines for the module $M_2|A_1$. Description of the four investigations suggested in the guidelines were given in Chapter one. A selection of thirteen more investigations for this module has been produced in this study. A description of them is given in Table 2.6.

Table 2.6

Investigation	General Content
1. Investigation of the history of $\sqrt{2}$.	Approximations of numbers, errors, geometrical constructions of irrational numbers.
2. The best way of calculating $\sqrt{2}$.	Functions, evaluation of formulas, error algebra, graphs.
3. The cheapest tin of apples.	Modelling process, area, volume of three dimensional shapes, proportion.
4. Graph and linear functions.	Evaluation of functions, modelling, graphs and interpretation of them, approximations, estimations.

Table 2.6 (contd.)

Investigation	General Content
5. Investigation of the curve which passes through the vertices of a specific series of equilateral triangles.	Geometrical properties of parabolas, algebraic patterns, properties of equilateral triangles, interpretation of graphs.
6. Parabola-Sine curves.	Constructions of conic sections, relation of parabola and cone, graphs of sine curves $y = a \sin x$.
7. Building the graph of $y = ax^2 + bx + c$ from the graph $y = x^2$.	Graphs of quadratic functions, vectors, transformations, dilations.
8. Solution of the equation $\sin x = \frac{x}{100}$.	Graphs of circular and linear functions, interpretation of graphs, approximations.
9. Investigation of the shortest route of a fly to move round a cone.	Trigonometric ratios in triangles, properties of cones.

Table 2.6 (contd.)

Investigation	General Content
10. Where the rugby ball should be placed to maximise the chance for a successful goal kick.	Trigonometric ratios, sine rule, approximations, graphs.
11. How long is an up and down hill walk?	Vectors, graphs, gradient, linear interpolation.
12. Trigonometric value of angles $\frac{\pi}{2^n}$ $\frac{\pi}{3 \cdot 2^n}$ without calculator.	Radian, trigonometric ratios, Pythagoras' theorem, properties of shapes.
13. Working out the values of $\sin \theta^\circ$ using the trigonometric values of 10° , 1° , $\left(\frac{1}{10}\right)^\circ$, $\left(\frac{1}{100}\right)^\circ$.	Trigonometric ratios, properties of triangles, parallel lines, Pythagoras' theorem.



The description sheets for each investigation are included in Appendix J. The content which is described for each investigation is not always fixed. It depends most of the time on the student's method of tackling the problem. This is the case particularly with open-ended problems. An attempt has been made to use different contexts so as to meet the interests of the individuals. Some, for example, the investigations 3, 10 and 11, show applications of mathematics to practical problems. Some others use concrete materials, like investigation 6 from the H.R. Jacobs' book which constructs conic sections. Some others are pure mathematical problems (investigations 5, 7, 9, 12 and 13), while others (investigations 2 and 8) are about numerical methods. There are also three book-based investigations, 1, 4 and 6, which attempt to develop in the students good study skills and the ability to collect information. Most of the mathematical processes which have been described earlier can be practised by this selection of investigations.

Searl (1984) suggests that investigational work needs to take place once per fortnight in the module. The students need to submit a report where they describe their work in the investigation. This report will be corrected by the teacher and supportive comments will be written on the script. The teacher and the students choose from the suite the investigations which are more appropriate for them.

A teaching guide for the investigations is included in Appendix J. This contains some possible solutions of the investigations and attempts to help the teacher in this new approach.

Learning/teaching approaches in the classroom.

It is the use of all these resources which can create an effective learning environment not just the use of particular materials, neglecting the contribution of the others.

In addition to these, other resources are also necessary in the classroom. Use of books, notes, worksheets, problem solving activities play an important role in learning mathematics. Assessment procedures are also needed so that the students obtain suitable feedback for their work.

The organisation of all these resources by the teacher is not an easy task. He or she must take into account the desires of the students and together choose the most appropriate activities.

This multimedia approach can meet the different cognitive and personality characteristics of the students. Mathematics is not just skills or facts, it is also concepts, structures, processes, attitudes. All these aspects have to be developed.

The environment in the classroom also needs to be appropriate for the most effective use of the resources. Practical work or creative discussion for example cannot easily take place in a formal environment. NCTM (1980), suggests that teachers should use diverse instructional strategies, materials and resources, such as individual or group work, well planned use of media, provision of situations that provide discovery, inquiry and basic skills, use of manipulatives, cyclic review of past topics, materials and references outside the classroom.

In the next chapter the description of the use of the materials in the classroom and their evaluation will be given.

CHAPTER THREE

EVALUATION OF THE MATERIALS

The approaches which have been adopted for the evaluation of the materials are those suggested by Parlett and Hamilton (1972). The main aim of the 'illuminative evaluation' which they suggest is to describe and interpret rather than to measure and predict.

"The aims of illuminative evaluation are to study the innovatory program: how it operates; how it is influenced by the various school situations in which it is applied; what those directly concerned regard as its advantages and disadvantages; and how students' intellectual tasks and academic experiences are most affected. It aims to discover and document what it is like to be participating in the scheme, whether as teacher or pupil; and, in addition, to discern and discuss the innovation's most significant features, recurring concomitants and critical processes".

The methodology in illuminative evaluation is not standard but it depends on many factors. The problem itself defines the methods used and not vice versa. 'Illuminative evaluation is not a standard methodological package but a general research strategy.'

The investigators try to present reality as it is or at least as they see it. They make no attempt to manipulate, control or eliminate variables but they take as given the complex scene they encounter. Usually methods which can be used include observations, interviews, questionnaires and tests; documentary and background sources.

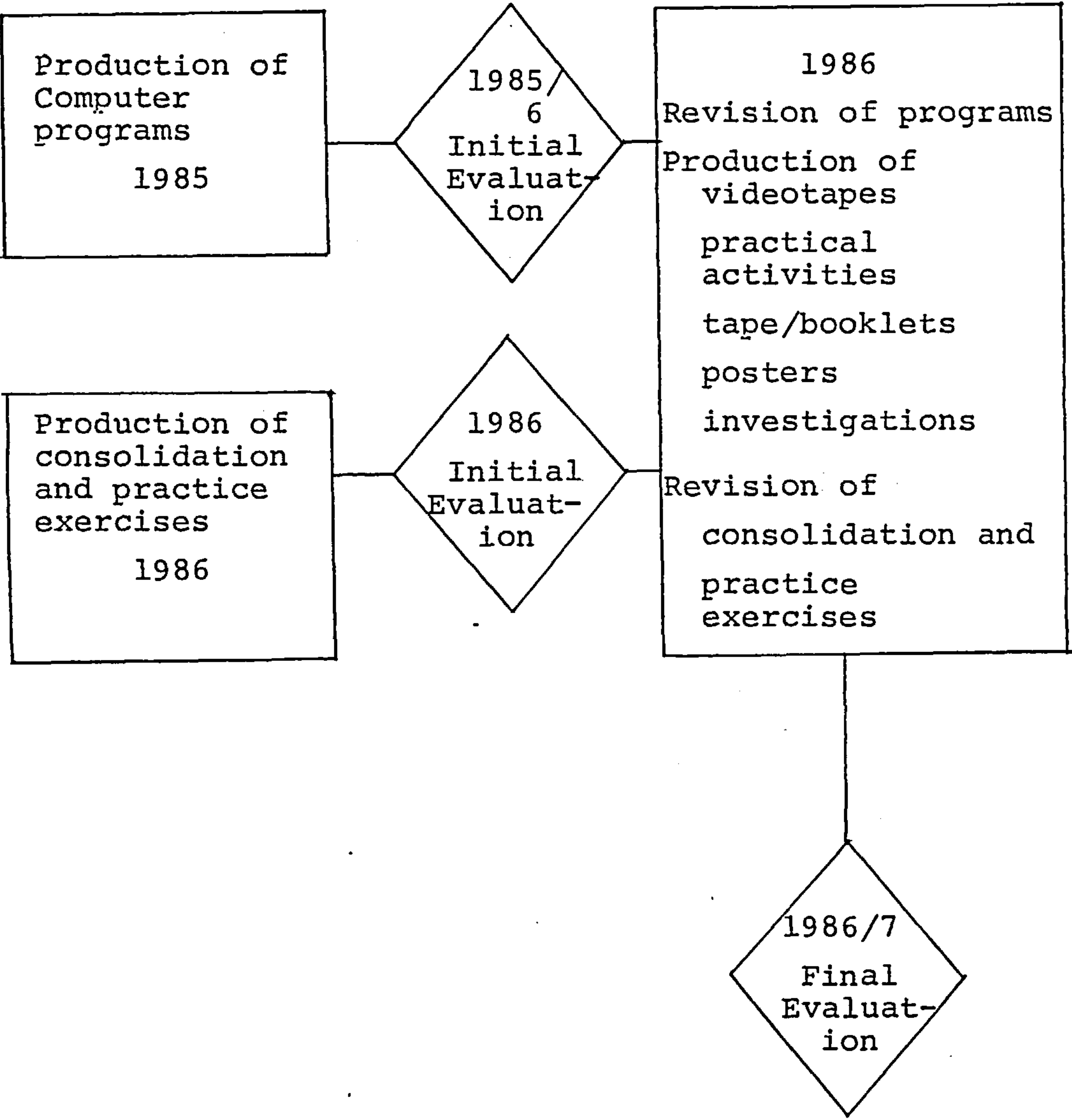
In the initial phase of the evaluation, the computer programs were used in five colleges of further education. The information was collected by questionnaires, short tests, classroom observation and short visits. The consolidation and practice exercises were evaluated in three schools using questionnaires. This initial phase resulted in a revision of these materials.

In the second phase, all the resources were evaluated in five colleges of further education and one school. The evaluation of the individual resources and of the whole learning environment at each institution by using classroom observation, questionnaires, and interviews is described systematically in the later part of this chapter.

This pattern of development and evaluation is shown in figure 3.1.

Figure 3.1

The pattern of evaluation and development of the materials.



Initial phase of the evaluation: Computer programs

The evaluation of the materials started in September 1985. The first version of the eight computer programs and of their associated worksheets were tested in five colleges. The main difference between these programs and the final versions described in Chapter two is in the presentation. These materials were tried out over about eight weeks, the standard duration of the teaching part of the module $M_2|A_1$.

The main aims of this evaluation were: to find any mistakes in the programming of the computer programs; to find parts of the programs which caused problems for the learners (difficult to use, unclear instructions, unattractive presentation, vague presentation of concepts); to examine the role of the computer programs in the learning environment; to examine the appropriateness of the worksheets associated with the computer programs. The evaluation would also familiarize the author with the learning environment, with the problems which lecturers and students encountered with the new approaches suggested in the module descriptors and in the guidelines.

Information was obtained using questionnaires for lecturers and students, short tests for students, completed worksheets for the computer programs, regular observation of the teaching-learning process in one college, short visits to three other colleges, informal discussions with lecturers and students.

The short tests tried to assess if the students had assimilated the teaching points of the computer programs. The questionnaires were the same for all the programs. Copies of the questionnaires and short tests are included in Appendix B.

Table 3.1

The program 'Bridge' was not evaluated. It is appropriate for the last week of the module and the lecturers did not use it through lack of time. The results from the students' questionnaires are shown in table 3.1, 3.2, 3.3.

Number of Students							
Program	Errors	Minimum	Line	Quadratic function	Cosine-sine curves	Radian	Vectors
very interesting	13 (10%)	3 (4%)	5 (8%)	4 (6%)	1 (5%)	1 (5%)	3 (23%)
interesting	99 (79%)	67 (88%)	56 (89%)	58 (84%)	15 (79%)	19 (90%)	10 (77%)
not interesting	14 (11%)	6 (8%)	2 (3%)	7 (10%)	3 (16%)	1 (5%)	0 (0%)
Worksheet							
very easy	30 (24%)	3 (4%)	14 (22%)	13 (19%)	0 (0%)	3 (14%)	3 (23%)
easy	83 (66%)	50 (66%)	48 (76%)	55 (80%)	12 (63%)	18 (86%)	10 (77%)
difficult	12 (9%)	22 (29%)	1 (2%)	1 (1%)	4 (21%)	0 (0%)	0 (0%)
very difficult	1 (1%)	1 (1%)	0 (0%)	0 (0%)	3 (16%)	0 (0%)	0 (0%)

From the figures in Table 3.1 it seems that most of the students found all the computer programs interesting although a number of students found the programs on cosine-sine curves, errors and quadratic function 'not interesting at all'. They also found the worksheets easy, except the worksheet on cosine-sine curves.

In Table 3.2 the difficulties reported by the students are listed.

Table 3.2

<p><u>"Errors"</u>: 49 students (40%) reported difficulties.</p> <p><u>In the program</u>: difficult to understand: the main teaching point, answers given in scientific notation, calculation of extreme values where the answer lies. Difficult to read the screen text.</p> <p><u>In the worksheet</u>: not enough space to record the numbers, more explanation.</p>
<p><u>"Minimum"</u>: 40 students (53%) reported difficulties.</p> <p><u>In the program</u>: difficult to understand, change and simplify the formulae.</p> <p><u>In the worksheet</u>: not enough space to record the formulae.</p>
<p><u>"Line"</u>: 10 students (16%) reported difficulties.</p> <p><u>In the program</u>: difficult to understand error interval, insert experimental data, copy graph from screen.</p>

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Table 3.2 (contd.)

<p><u>"Quadratic function"</u>: 14 students (20%) reported difficulties.</p> <p><u>In the program</u>: unfamiliarity with computer, difficult to change x and y domain if graph was out of screen.</p> <p><u>In the worksheet</u>: a lot of work to do.</p>
<p><u>"Cosine-sine curves"</u>: 7 students (37%) reported difficulties.</p> <p><u>In the program</u>: difficult to understand it.</p> <p><u>In the short test</u>: difficult to draw $y = \sin x + \frac{1}{2} \cos 2x$.</p>
<p><u>"Radian"</u>: 2 students (9%) reported difficulties.</p> <p><u>In the program and worksheet</u>: difficult to use the program to complete the last table of the worksheet.</p>
<p><u>"Vectors"</u>: 2 students (15%) reported difficulties.</p> <p><u>In the program</u>: difficult to load the program each time they wanted to see it.</p>

In Table 3.3 the comments of the students are summarised.

Table 3.3

Program	Comments
Errors	<p><u>About program:</u> other parts of error algebra to be included, illustration of the calculation of the extreme values, less complex explanations, better presentation using colour, graphics, sound, better way of seeing the program again instead of typing 'RUN', remind students to type <return> after entering a number.</p> <p><u>About worksheet:</u> more space available for recording numbers.</p> <p><u>About course:</u> more practical applications of errors, more use of computers, more individual help from lecturer. Some found it interesting and reasonable, others hard.</p>
Minimum	<p><u>About program:</u> better explanations of formulae, better layouts of formulae, more accuracy for the turning point, more interesting screen displays. Monotonous, too long, what the point is of guessing from graph.</p> <p><u>About course:</u> more examples to be provided in the lesson, individual use of computers, interesting as it is related to practical problems, difficult to understand.</p>

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Table 3.3 (contd.)

Program	Comments
Line	<p><u>About program</u>: explain error interval, more participation of the user in the calculations, better presentation (less text, colour), give option for changing input.</p> <p><u>About worksheet</u>: more questions to be solved.</p> <p><u>About course</u>: very geometrical, hard, interesting</p>
Quadratic function	<p><u>About program</u>: better presentation (colour, clearer graphics, correct line ending), print the coordinates of turning point, draw the point if coordinates are supplied, give option for seeing the program without typing 'RUN'.</p> <p><u>About worksheet</u>: less work.</p> <p><u>About course</u>: easy, interesting, so much time spent, use of calculus, to find turning points, useful.</p>
Cosine-Sine Curves	<p><u>About program</u>: clearer instructions, better explanations and simplification of the context of the program.</p> <p><u>About course</u>: easy, after doing this program difficult.</p>
Radian	<p><u>About program</u>: better presentation (colour, graphics), more cases of angles in the last table.</p> <p><u>About worksheet</u>: clearer instructions.</p> <p><u>About course</u>: interesting.</p>

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Table 3.3 (contd.)

Program	Contents
Vectors	<u>About program:</u> they liked the use of colour, an option to load the program automatically.

The worksheets associated with the computer programs revealed the inadequate grasp of some mathematical points, for example, not understanding answers of the form 1.28 ± 0.007 or the difference between numbers like 3 and 3.00; problems in estimating from graphs; relation of radians and degrees (some wrote 1 radian = 180 degrees). Generally, the students completed the worksheets successfully.

The results from the short tests are shown in Table 3.4. The tests were completed by the students after using the relevant computer programs. Some had to be completed using the programs while others did not. (Some students used the programs in cases where they should not have used them.

Table 3.4

Program	Comments on the answers to the short tests.
Errors	About half of the students correctly answered all the questions of the test. The most common mistakes were about absolute error bounds of negative numbers (eg. absolute error bound of -0.5 is -0.05) and of decimals with zeros at the end

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Table 3.4 (contd.)

Program	Comments on the answers to the short tests.
Errors (contd.)	(eg. absolute error bound of 4.500 is 0.05). They also had difficulty in calculating the correctly rounded number.
Minimum	Most students could estimate the turning point from the graph but they could not find a method to approximate it to two decimal places using their calculator.
Line	Most students drew the graph of the straight line by copying it from the screen. Most could not find the error interval.
Quadratic function	The students did not have any particular difficulty. A misunderstanding appeared between "roots" and "points" where the graph cuts the x-axis. Some had difficulty in expressing a point in terms of its coordinates.
Cosine-Sine curves	Students found it difficult to sketch the graph of $y = \sin x + \frac{1}{2} \cos 2x$ using the graphs of $y = \sin x$ and $y = \cos 2x$.
Radian	Their difficulties were not clear because they used the computer program to complete the test.
Vectors	Problems in adding non-consecutive vectors.

Seven lecturers were involved in this evaluation. Teaching guides or instructions were not given to them in this first trial. Most lecturers responded to the questionnaires. From those programs which were used in some colleges, questionnaires about the program 'Vectors' have not been answered. In the questionnaires some questions were about the materials while others about the course itself. All the lecturers reported that the students liked the programs. All the students in the class used them mainly as reinforcement materials. The number of students in the class was about 15 to 20 although there were classes with 7 or 10 students. The materials used in the course were notes, worksheets, exercise sheets, computer programs, calculators. All the lecturers assessed the students' performance from their workfile, their performance in worksheets, and in the final test. The main learning approaches which the lecturers used were exposition, discussion, practice of skills and problem solving. The practical and investigational work were neglected. Some of the lecturers' responses on the individual programs are summarised in Tables 3.5 and 3.6 .

Table 3.5

	Errors	Minimum	Line	Quadratic function	Cosine-Sine curves	Radian
Difficulties of students	Choose the correctly rounded number and understand some error terms.	Understand the algebra, formulae and transfer the formulae from screen to paper.	Understand the error interval.	No difficulties	Understand the initial problem from mechanical and mathematical point of view	No specific-ation of the difficulties.
Time spent on program and worksheet.	From 30 minutes (in most cases) to 1½ hours.	From 15 minutes (in most cases) to 1½ hours.	From 15 minutes (in most cases) to 45 mins.	From 15-30 minutes (in most cases) to 1 hour.	About 1 to 1½ hours.	About 15-20 minutes.

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Table 3.5 (contd.)

	Errors	Minimum	Line	Quadratic function	Cosine-Sine curves	Radians
How the students found the worksheet.	1 wrote 'very easy'. 2 wrote 'easy' 3 wrote 'OK' 1 wrote 'very difficult'	2 wrote 'easy' 2 wrote 'OK' 1 wrote 'very difficult'	2 wrote 'easy' 2 wrote 'OK'	5 wrote 'easy'	1 wrote 'easy' 1 wrote 'very difficult'	1 wrote 'very easy' 1 wrote 'easy'
Other topics which could be taught by the computer in that week.	Errors in addition, subtraction, products, roots and powers. Practical examples.	Trapezoidal rule, production cost-profit using graphs.	Illustration of the graph $y = mx + c$.	Trapezoidal rule, axis of symmetry.	No suggestions.	Solving right-angled triangles, sine-cosine rules, area of triangles.

Table 3.6

Program	General comments and suggestions of lecturers.
Error	The program can only be used as reinforcement material as not being comprehensive enough. Pictorial explanation of the correctly rounded number, use of colour, explanation of the extreme values and of answers like 1.28 ± 0.007 . Extend to include errors in multiplication, addition, subtraction, power, roots.
Minimum	Useful especially as an extension material but time consuming. Less complicated problem, less complex formulae and more time to be spent on the final graph. Use of colour.
Line	A good short package, good for either investigation or for checking results of questions already done mechanically. Suitable for illustration. Insufficient time to analyse the program. The second example of the worksheet not so applicable for linear interpolation.
Quadratic function	Very good especially for students of limited ability. The best so far. Useful for illustration and for self-learning. Useful for sketching curves and finding zeros. Extend to include axis of symmetry, min/max turning points. Improve presentation.

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Table 3.6 (contd.)

Program	General comments and suggestions of lecturers.
Cosine-Sine curves	The module descriptor does not state clearly what is required in the area of trigonometry.
Radian	Students did not appreciate the relationship that radian measure is the arclength divided by the radius because the circle was of unit radius.

Results from observations

The author was attending three classes of two different lecturers... There were about 20 students in each mixed ability class. Some students were full-time (in two of the classes), others were part-time but they were all following the same course.

The students mainly used the computer programs in the computer room. This sometimes caused problems because it was occupied by other classes. In the computer room there were about 8-10 BBC microcomputers and the students worked in pairs or in groups of three. Generally, they enjoyed working with the computer programs, as being something new for them. They expected to use the computer for 'games'. They had difficulties in concentrating on the screen and needed help from the lecturer to appreciate the main points of the programs. Mostly, the programs were used as reinforcement materials after the introduction of the relevant topic. Most teaching points, like finding the correctly rounded number, fitting a straight line to experimental data, period of circular functions were encountered by the

students for the first time at the computer programs. The students had difficulty in understanding the explanation about the correctly rounded number and the lecturer suggested to them representing the numbers on a number line. In 'cosine-sine curves' program, difficulty was encountered in understanding the mechanism illustrated. They also had difficulty in estimating the optimal can diameter in the 'Minimum' program. Some students who had done Higher Mathematics could not appreciate the reason for estimation, having been trained to use calculus.

The main teaching approach was exposition and emphasis was given in covering the content of the module neglecting other approaches such as problem solving, investigational and practical work. The main reason for this was the 'lack of time': a view supported by all the lecturers. There was also discussion with the students but it was very limited. Some notes as handouts, or dictated, were given to the students. The assessment worksheets used in the course were those suggested in the guidelines. One lecturer used them as a test while the other as the main resource in the class, tackling the examples with the students on the board or helping them individually. The lecturers' attitudes about the computer programs were not clear but certainly they did not regard them as an essential element of the course, possibly because they were not familiar with this approach.

Results from short visits

The situation in the other colleges was somewhat different to judge from a short visit.

The computer programs were used in the classroom using one or two microcomputers. Some students worked in small groups on the computer programs while others carried out individual work.

Most of the classes had about 20 students but some were smaller. The students' background was more uniform and all had either O-grade Mathematics or they had completed the module G₃.

The main teaching approach was exposition, although practice exercises and problem solving were used. Practical or investigational work were not really used, at least not in the form suggested in the guidelines. Some practical examples were, however, used.

The lecturers were confused by the approaches suggested for the module. It was not clear to them from the module descriptor what they had to include in their course, how to approach it and how to assess it.

Decisions about changes

This first evaluation of the computer programs helped the author to identify points in the programs which did not work well in the classroom. Changes in the presentation of the programs were made so that they were more attractive but also to give clearer explanations of the concepts. For example in 'Errors' a pictorial explanation of the correctly rounded number was adopted, in 'Minimum' a better statement of the practical problem and of the formulae, in 'Radian' and 'Cosine-Sine curves' changes in the way of presenting the concept of arclength and of the mechanical problem. In all of them colour was added.

The evaluation also revealed a picture of the learning situation which helped the author to decide what was missing in the course and to develop other teaching/learning materials.

Initial phase of evaluation: consolidation and practice exercises

In spring 1986 the four booklets on the consolidation and

practice exercises were tested with secondary school pupils who were taking O-grade or Standard Grade examinations in mathematics that year. Three schools helped in this evaluation. The results from a test on three sets of exercises 4, 14, 15, which was given to the top set of the Second Year in another school, were also considered.

The mathematics syllabus which these students had met was similar to the students' of module $M_2|A_1$.

The main aims of this evaluation were to find any mistakes in the exercises; test the level of difficulty; test the appropriateness of the target time; assess any improvement in the students' attainment; test the appropriateness of the content and their effectiveness for diagnosing students' ignorance.

Information was obtained using questionnaires for students and teacher (see Appendix B). In the students' questionnaires there were tables for recording their performance in each set of exercises. There were no responses on the questionnaires for booklet 4, and few for booklet 3 (4 students replied).

Table 3.7 shows the average success rate of the students and the average time taken for each set of exercises. The target time of each set is also shown.

Table 3.7

Booklet 1

Exercise	1	2	3	4	5	6	7	8	9	10
Number of students	55	76	76	90*	75	72	71	71	67	67
Success rate	83%	86%	70%	72%	78%	83%	89%	82%	90%	88%
Average time (mins)	10	10	8	7	6	8	7	9	7	8
Target time (mins)	10	10	5	5	4	6	5	8	6	7

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Table 3.7 (contd.)

Booklet 2

Exercise	11	12	13	14	15	16	17	18	19	20
Number of students	51	51	48	73*	65*	38	42	42	27	27
Success rate	88%	88%	74%	81%	71%	86%	76%	86%	80%	86%
Average time (mins)	6	7	13	11	8	7	8	11	8	6
Target time (mins)	5	6	12	9	6	6	6	10	6	5

* 25 from those students are those who took the test on three sets of exercises. Their average time is not taken into account in the table.

From the table it can be seen that overall the students could do most questions in each set of exercises but the time which they spent was generally longer than that suggested. A relatively low success rate appeared on exercises 3 (operations on integers and rationals), 4 (square roots, indices, scientific notation, 5 (rounding of numbers), 13 (area of three dimensional figures), 15 (area and volume of similar figures), and 17 (exact values of trigonometric values of 0° , 30° , 60° , 45° , 90°). Results from the test in one school showed the diagnostic role of the exercises. For example, the most common mistakes in indices were of the form $2^0 = 0$, $5^2 + (-5)^2 = 0$, $(-3)^3 = 27$, $10^5 : 10^2 = 10^7$, $1 - 3^2 = 1 + 9$. In the area and volume of similar figures the relationship of ratio of volumes and ratio of sides was found difficult.

Table 3.8 shows the students' views of the difficulty and usefulness of the exercises.

Table 3.8

	Number of students	Very easy	Easy	OK	Hard	Very hard
Booklet 1	69	4 (6%)	13 (19%)	43 (62%)	8 (12%)	1 (1%)
Booklet 2	39	0 (0%)	4 (10%)	22 (57%)	11 (28%)	2 (5%)

	Number of students	Helped a lot	Helped a little	Helped not at all
Booklet 1	70	3 (4%)	62 (89%)	5 (7%)
Booklet 2	39	4 (10%)	30 (77%)	5 (13%)

From the table 3.8, it can be seen that most students found the exercises appropriate to their needs and helpful.

Their comments on booklets 1 and 2 are summarised below:

Helpful because they identify particular problems, practice of basic work, cover wide range of questions; give confidence, remind them of things they have forgotten; good for revision; target time too short; some questions hard; some not relevant to what they had been taught; some misprints; uninteresting and boring; they should be done daily, not in groups of three and four per day.

Two teachers answered the questionnaires and both found the exercises helpful. One teacher commented that these types of exercises were not familiar to her pupils who were mainly doing problem solving activities and they were quickly bored by them. Some topics, for example, vectors, were not in the Standard Grade Course. The booklets could be used for revision but she would prefer more 'real-life' applications of the skills. She found them more relevant to O-Grade Mathematics.

In the light of this pilot study some changes were made in the booklets. The target time of each exercise was changed to make them suitable for the majority of the

students. Some misprints were corrected and, in the set about the indices (booklet 1, exercise 4), two or three exercises were simplified further.

The effectiveness of the exercises could not be examined in more detail because the situation did not allow for more detailed comparison.

Final evaluation of the materials

The evaluation of all the materials started in September 1986 in five colleges and one high school. Four of the colleges had helped with the initial evaluation of the computer programs.

The teachers were sent a document which described the main features of the materials. Some, who had participated in the first evaluation, already knew something about the materials from informal discussions which they had with the author. The participants were asked to choose the materials which they would like to use in their classroom. Some of the materials were not available when the evaluation started and they were sent during the evaluation process. At this stage the teaching guide for the practical activities was incomplete. Table 3.9 shows the materials requested and used by each institution, as well as the number of those supplied.

Table 3.9

	College A			College B			College C			College D			College E			School		
	R	S	U	R	S	U	R	S	U	R	S	U	R	S	U	R	S	U
	e	u	e	e	u	e	e	u	e	e	u	e	e	u	e	e	u	e
	q	p		q	p		q	p		q	p		q	p		q	p	
	u	e		u	e		u	e		u	e		u	e		u	e	
	s	i		s	i		s	i		s	i		s	i		s	i	
	t	e		t	e		t	e		t	e		t	e		t	e	
	e	d		e	d		e	d		e	d		e	d		e	d	
Computer programs	✓	8	3	✓	8	8	✓	8	8	✓	8	8	✓	8	8	✓	8	8
Video-tapes	✓	31	2				✓	31	0	✓	31	31				✓	31	some
Consolidation booklets	✓	4	1				✓	4	0	✓	4	4	✓	4	4	✓	4	4
Practical activities	✓	10	1				✓	11	0	✓	15	11	✓	11	3	✓	11	9
Tape/booklets	✓	5	0				✓	10	0	✓	8	8				✓	10	10
Posters	✓	4	2				✓	4	0	✓	4	4	✓	4	4	✓	4	4
Investigations	✓	13	2				✓	13	0	✓	13	4	✓	13	0	✓	13	3

There was not any feedback from college B and college C. In college B the teacher appreciated the improvement of the computer programs but he was involved in a different module so he did not use the new version. In college C

they used only the computer programs because of 'lack of time' and of accommodation problems in the college.

Information was collected by regular observation of the classroom in one college (college A), short visits near to the end of the module to the other institutions, interviews, discussions, questionnaires for students and teachers about the practical activities, written reports about the investigations, and questionnaires about the whole learning environment at the end of the module. Copies of the questionnaires are included in Appendix C. The interviews aimed to make clear some points of the questionnaires and to give more information about the students' background, their interests, attitudes, difficulties, and future plans. A plan of the questions which the students would be asked had been prepared but sometimes other points arose from the discussions. The interviews were tape-recorded and transcribed in college A and college D. In the other institutions the conditions allowed only informal discussions with the class as a whole (college D) or with some individuals (school). The students were interviewed in pairs. In college A the interview did not take place in the classroom, while in college D the students were interviewed while they were working. The interviews lasted about 10-15 minutes for each group. The teachers were not interviewed but only informal discussions took place. Information on investigations was obtained by introducing a competition involving four of the investigations. A book token for £5 was given to the student with the best report on each of the four investigations. The instructions for the teacher are included in Appendix C.

The main aim of the evaluation of the materials was to show the role of each in the learning of mathematics and their interrelationship in the learning environment.

Results from college A

The materials were used by a class of twelve day release students. The teacher had been involved in the first evaluation of the computer programs.

Computer programs: The students used them in the computer room working in pairs or in groups of three. The teacher helped them understand the main parts of the programs. There was discussion between the students and between the students and the teacher. The students could understand satisfactorily the main teaching points of the programs. The attitude of the teacher towards using computer programs was more positive than during his previous involvement with the computer programs. He also used once a short program about evaluation of formulae. The students spent about 30 to 40 minutes working on each program and it seemed that they enjoyed it.

Videotaped expositions: The teacher had to bring the video equipment into the classroom and with the students' help he managed to operate it. He used the two first programmes on quadratic functions. He had not introduced the topic before. After watching the first programme the teacher suggested an alternative method. Some students did not follow the programme. The next one was not completed because the teaching period finished. The teacher liked these materials and found it helpful to see different teaching approaches.

Consolidation and practice exercises: The teacher gave booklet 1 to the students without any explanation on how they could use it. Discussion with the students showed that most of them did not use the booklets.

Practical activity: Activity 8 (ice cubes) was used in the classroom. Four students did the activity in a very formal environment after completing the assessment worksheet of that week. The teacher did not help them or discuss the activity with them as he was helping the rest

of the students to complete the assessment worksheet. The students did not have any particular difficulty in completing the activity but because of lack of appropriate support they did not seem to appreciate the mathematics involved in it. Because of the classroom setting, desks in rows, and lack of cooperation between the students who were doing the activity and between the teacher and the students, the activity did not seem to be effective. From the questionnaires which the students completed it seemed that most of them enjoyed the activity and they did not have any problem with the instructions on the description sheet. The time spent on it was about 30 to 50 minutes. From their comments it appeared that they had some problems because the ice cube melted in a non-uniform way. The teacher, in his questionnaire, suggested that a suitable laboratory is needed otherwise a lot of time is wasted.

Posters: Two posters, 'Errors' and 'Area under curves' were placed on the wall beside the blackboard. The teacher did not encourage the students at all to look at the posters and no discussion was created in the classroom.

Investigations: The teacher introduced the first competition investigation which was about the calculation of the square root of two. He told the students that it was not compulsory to do it but it would help them with mathematics. He made some suggestions about the presentation of the report and what questions they could consider. He told them that he could help them if they had any difficulties. Two students tried the investigation but only one wrote a report. The other tried and asked for help from the lecturer and the author, but he did not complete it because, as he said, he found it difficult and he did not have enough time. From the student's report it appeared that he had understood the process of iteration. He used mainly

graphical methods to tackle the problem. The presentation of his work was clear but his graphs could have been improved by using graph paper or flexicurve. Comments were made on his work, reinforcing his attempt and offering some suggestions at points of the investigations which could have been improved. He was the only student who undertook the second investigation, the design of the milk carton. It was clear that he took into account the suggestions made in the previous investigation. He also enjoyed it very much as he commented. This activity brought to light some of his misconceptions, his way of thinking and organising information, his mathematical background and certainly his ability in mathematics. Most students did not see the importance of this activity as they were not being particularly encouraged by their teacher.

The tape-booklet sequences were not introduced to the students at all because, the lecturer stated, there was not a cassette player available in the college.

The main approach of learning was exposition by the teacher. Some notes were dictated or handed out to the students and the main work of the module was based on the assessment worksheets. The practical and investigational work were very new methods for the teacher and the students. Because of this, the short time available and the traditional setting of the classroom, the teacher could not organize the use of the materials properly. The teacher stopped using the materials five weeks before the end of the module because he did not have time to cover the rest of the module's content.

Results from the final questionnaires: Eleven of the twelve students in the class completed the questionnaire. Most wrote that they liked the course. One wrote that it helped him to understand things which he had not understood at school, while another wrote that he had learned

more about computers. One found the course boring. Almost all found the course useful for their future studies. Most students wrote that they had noticed an improvement in their mathematical ability and that they liked mathematics more after the course than before. They also had friendly relationships with their lecturer. How they found the various topics of the module, what materials they used and how they found them is summarised in Tables 3.10 and 3.11.

Table 3.10

	Topics				
	Errors	Linear funct.	Quadrat- ic funct.	Trigon- ometry	Vectors
Very difficult	0	0	0	0	0
Difficult	2	0	2	0	0
Average	8	8	6	10	7
Easy	1	2	2	0	4
Very easy	0	1	1	1	0

Table 3.11

	Computer Programs	Practical Activities	Consolidation exercises	Posters	Videotapes	Investigations	Tape Booklets	Books	Notes
Number of users *	11	4	1	2	9	7	-	3	9
Very interesting	1	0		0	0	1		0	0
Interesting	10	7	-	2	3	5	-	2	8
Not interesting	0	1		0	4	2		1	1
Very helpful	1	0	0	0	1	1		0	2
Helpful	10	6	0	1	4	3	-	3	4
Not helpful	0	1	1	1	4	2		0	0

* The discrepancies between those who used the materials and those who made comments on them are either because students did not complete the questions carefully or because they saw others use them, for example in practical activities.

From the tables it seems that the students found the topics of the module of average difficulty, although some found the Errors and the Quadratic functions difficult. They found most of the materials interesting and helpful, except for the video programmes. The students had 0-grade mathematics while one who did also the investigations had Standard Grade 1. The main problem which they had with the course was that they did not have enough time to go into particular topics in depth. One reported that he had difficulties catching up with the work because he did not start the course at the beginning, while another said that he had not done vectors before. The main change which they would like in the course was to have more time available.

The teacher's questionnaire shows that he feels that the students are cooperative and he has a very good relationship with them. The time allocated for different activities per week (3 hours) was: 45 minutes for the computer programs; 5 minutes for practical activities; 2 hours and 5 minutes for exposition and 5 minutes for investigations. He commented that if more time were available he would have spent it on the other activities rather than on exposition. The main problem which he had with the materials was 'lack of time'. Computers and experimental equipment were not available in the classroom but could be obtained on request. As he reported, the computer programs were used as reinforcement materials and the students liked them. He noticed that the students liked the practical activity and the videotaped exposition. He found the video taped exposition very good but lack of time prevented further work. The investigations required too much work of the students in the short time available. He thought the computer programs and the video programmes were the most successful in the classroom. He believed that more equipment in the class would help. "Computer, overhead projector, laboratory equipment, as proposed

in the suggested layout for a maths classroom suitable for modular instruction." He appreciated the attempt to assist him in his work but because of lack of time he was unable to investigate all the materials.

Results from interviews: Eleven students were interviewed. The interviews gave some information about the students' background. They were working as apprentices and they had a day release. The module had been chosen for them by their employers. Almost all had no previous experience of college life and they preferred life in school, although some commented that they could speak more to the teachers in college than in school. The interviews gave also some information about the students' attitudes towards the course and of the individual elements of it. They liked the course but their main problem was the limited time in which they had to cover too many things. Mostly they preferred to work in groups because they were sharing ideas, but one found it distracting and another found that he worked faster working by himself. They did not use books for their studies except for one who used some for the investigations and another to clarify some parts of the module. The most difficult topic which they found was about Errors and they did not seem to appreciate its practical importance. They found the assessment worksheets of average difficulty. They liked mathematics although some of them found it difficult. They liked the computer programs. One commented "It helps you understand what you have done in the class." One student found the worksheets associated with the computer programs very structured and he would prefer the questions to be more open. Most did not do the consolidation and practice exercises. One commented about practical mathematics "It helps you to understand by yourself what you are doing and why. You don't just see the graph until you actually do the graph."

One found the practical activity with the ice cube 'not very practical'. He added that he did not find it very helpful and it was more strain to do it. Some of them found the idea very interesting. One said, "It depends on the person. If you can visualise as you write down, you don't need practical work, but if not, doing practical work and written work together you can understand why". The main comment was that this type of activity occupies a lot of time. They did not like the videotaped expositions mainly because they found them complicated. One said "too deep in the subject". Some commented on the different approach in the programmes from those that were used in the class. Most students did not try the investigations because, they claimed, of lack of time. The student who did the two of them found them interesting. Someone said that he could not do the investigation about the square root of two because he could not understand "what the point was". Most students had seen the posters but they did not pay much attention to them.

Results from college D

Most students who used the materials had already completed the module $M_2|A_1$ with main teaching methods worksheets, discussions, group problem solving. The main reason for this was that the materials were received by the college when the module had already started and there was a resistance to change. The teacher commented "our assessment is seen, we want more teaching", implying she and her colleagues wanted more time for exposition. The materials were evaluated mainly in a two-hours' session each week. The teacher believed that the ideal situation would be if the teachers had familiarised themselves with the materials prior to teaching the module and were able to introduce them topic by topic. Some important points are summarised below for each material as they appeared from discussion with the

teacher and from the questionnaires of the teacher and students and written comments of the students.

Computer programs: They were used as reinforcement materials. They gave opportunity for cooperation between students who were familiar with computers and those who were not. They helped students to learn or reinforce bits of information. The students were working mainly in pairs on a microcomputer which was in the classroom. Most students liked the programs. Some students who had not learned about 'Errors' before could not understand the relevant program, while most students had difficulty in understanding the program 'Cosine-Sine curves'.

Videotaped expositions: The use of video by the whole class for about one to two hours continually was not successful at all because the students became bored. Individual or in pairs use of the programmes gave better responses. The teacher found it good for her to see subjects 'taught' by someone else. She found them useful as an open learning resource. From students' comments it appeared that they had difficulty in concentrating for a long period on the screen. Some students said that they would prefer shorter programs while a mature student who had come to the college initially with 'no mathematics' at all, found them very useful. He commented "This video proved most useful to me because I could rewind and rewind so as to get the message into my head. Linear equations are more easily understood when you have the visual diagrams and in this video it was quite easy to follow".

Consolidation and practice exercises: The teacher tried to get some students to do an exercise each day but found them lacking in self-discipline. The students were willing to do them at the beginning of each mathematics session and found them very helpful. The teacher found them 'excellent' as they cover from each topic the key points.

Practical activities: As the teacher said, the practical work helped her to identify aspects of the students' abilities which she had not discovered before. In addition, sometimes the activity caused creative discussion. The students were working on the activities mainly in pairs. The teacher remarked that they all enjoyed this approach. The results from the students' and teacher's questionnaires on the practical activities are summarised below.

Practical activity 2 (whisky bottle): 2 students.

Time spent: about 75 minutes.

Students' responses: Interesting.

Teacher's responses: The students were completely absorbed in the activity. Worthwhile, giving practical insight to the students. Some small problems with the use of burette but it was replaced with an on/off tap. Difficulties in understanding 'increase in loss'.

Practical activity 3 (toilet roll): 5 students.

Time spent: From 10 to 90 minutes.

Students' responses: Most found it interesting. They had difficulty in measuring the length of toilet roll.

Teacher's responses: The students got bored. Worthwhile the time spent. Main problem the measurement of the length of the roll.

Practical activity 4 (density of metal): 2 students.

Time spent: From 15 to 60 minutes.

Students' responses: One found it interesting while the other did not. Container was leaky (from practical 9).

Teacher's responses: Students familiar with the experiment from physics classes did it quickly. Neither repetitive nor boring, worthwhile as conveying ideas on errors and units. More help might be given by indicating the units in the record sheet.

Practical activity 8 (ice cubes): 1 student.

Time spent: 35 minutes.

Students' responses: Interesting.

Teacher's responses: Modified the experiment by suspending the ice cube from a string.

Practical activity 9 (flow of water from holes):

2 students. Time spent: 60 minutes.

Students' responses: Interesting, but container was leaky.

Teacher's responses: Problems with equipment.

Practical activity 12 (overhead projector): 2 students.

Time spent: 10-50 minutes.

Students' responses: Interesting but difficult to obtain the parabola. More explanations on how to get the curves.

Teacher's responses: Students spent about 10 to 50 minutes but got frustrated with lack of definition with parabola and hyperbola.

Practical activity 15 (minimum of quadratic function):

1 student. Time spent: 60 minutes.

Students' responses: Not interesting, complicated instructions, too time consuming.

Teacher's responses: Students got fed up, not worth the time spent.

Practical activity 16 (quadrilaterals): 4 students.

Time spent: 20 to 35 minutes.

Students' responses: Most found it interesting. Difficulties in finding the maximum area.

Teacher's responses: Students seemed interested initially but got fed up with the repetition. A suggestion of an initial angle θ could help.

Practical activity 17 (cylinders): 5 students.

Time spent: 30 to 40 minutes.

Students' responses: Not interesting.

Teacher's responses: Generated no interest at all, not worth the time spent.

Practical activity 19 (radian measure): 5 students.

Time spent: 15 to 35 minutes.

Students' responses: Most found interesting. One suggested use of more durable discs, while another did not find any relevance to course.

Practical activity 22 (pin board): 7 students.

Time spent: 5 to 20 minutes.

Students' responses: Most found interesting. Some wanted clearer instructions. Some suggested more nails on the board.

Teacher's responses: More nails on the board.

Tape-booklet sequences: The teacher found them "excellent". They used them mainly in the classroom and some students wanted to borrow at home. They liked them very much. Most students found them very helpful.

Posters: The teacher commented that the students noticed them, read them but they did not make any comments. They helped to create a "maths" atmosphere but the students did not appreciate the content very much.

Investigations: As the teacher remarked, the students attempted to do some investigations in the classroom, completing their work in their own time but they had difficulty in seeing them as anything other than problems whose answer could be found in a short period of time. Initially they liked the idea but when they realised how much work was involved they quickly lost interest.

It appeared from the short visit that the teacher had a very friendly relationship with the students. All the materials were arranged around in the classroom and she negotiated with the students what activities they would do. One microcomputer and a tape recorder were in the classroom. A number of posters were on the wall; textbooks, tape booklets, consolidation and practice exercises were in various places in the classroom. The students were working in groups.

Results from the final questionnaires; Seven of the nine students in the class completed the questionnaire. Most students liked the course. One wrote "I did enjoy the course. It was interesting and the practical work helped you to put maths in everyday life", another student wrote "It gave us a break from the written work but I must admit I didn't really like the course because the practical took too much time for all you learnt out of it". Most found the course helpful for their future, and noticed some improvement in their ability in mathematics. They also liked mathematics more after the course. As they remarked, their relationship with the teacher and the other students was friendly. How they found the various topics of the module, what materials they used and how they found them is summarised in Tables 3.12 and 3.13 .

Table 3.12

	Topics				
	Errors	Linear funct.	Quadrat- ic funct.	Trigon- ometry	Vectors
Very difficult	1	0	0	0	0
Difficult	2	1	2	0	1
Average	2	4	3	3	1
Easy	0	1	2	2	4
Very easy	0	1	0	0	0

Table 3.13

	Computer programs	Practical activities	Consolidation exercises	Posters	Videotapes	Investigations	Tape Booklets	Books	Notes
Number of users*	3	6	2	3	3	3	2	2	2
Very interesting	0	0	0	0	0	1	1		
Interesting	1	5	2	3	2	1	1	-	1
Not interesting	2	1	0	0	1	1	0		1
Very helpful	1	0	0	0	0	0	1		
Helpful	0	3	2	1	3	1	1	-	1
Not helpful	0	1	0	2	0	0	0		1

* The discrepancies between those who used the materials and those who made comments on them are because some students did not complete the questions carefully.

From the tables it seems that the students found the topics of average difficulty apart from 'Errors' which they found particularly hard. They found most of the materials interesting and helpful. Most students had O-grade mathematics while one had Standard Grade, and one Higher Grade mathematics. Most did not have any particular problem with the course. One wrote that a textbook could be helpful. Another wanted better equipment and clearer instructions in the worksheets.

From the teacher's questionnaire, it seems that the atmosphere in the classroom was very friendly. The teacher had a lot of help from other departments and technicians in the college with the practical work. She found that the materials complemented each other and she could not say what was the most successful as the students had different preferences. Other materials she used were worksheets and some textbooks. The students enjoyed the course as it progressed and became more used to different teaching methods. They found 'Errors' a difficult topic. The students were of very mixed mathematics backgrounds but this could be overcome, because of the small class size, by individual tuition. The assessment of the course was according to the module descriptor but a need for assessment of practical work and individual thinking was indicated. The teacher would use selected parts of every kind of material in the future as the students benefit from having mathematics presented in a variety of ways. She used open learning materials from O-grade mathematics course and college-made posters. She thought ideas about teaching/learning could be disseminated by displayed materials and discussion. She suggested a 'mathematics workshop', more provision of money for microcomputers, tape-booklets, etc.

Results from interviews: A general discussion with the students in the classroom showed that the students enjoyed the course. They mostly liked the practical activities and some had appreciated the mathematical points of them while others had not. In the discussion about the poster 'Puzzle' they had difficulty in giving some explanations. A student who had done 'Higher mathematics' was very familiar with the computers. He particularly liked the 'Radian' and 'Quadratic function' programs while he found the 'Cosine-Sine curves' complicated. The teacher and students supported the view that the teaching point of the program 'Error' was very new for them. Some students had difficulty in knowing what to write down on the worksheet as they were running the program. Some parts of the content of the module did not seem relevant to their course (building engineers). Most did not like the video programmes.

Results from college E

The investigations, although they had been requested by the college, were not undertaken because of the limited time available. Some administrative problems and illness of the teacher caused some difficulties also in using the materials. The teacher had also participated in the initial evaluation of the computer programs. Some points are summarised below for each type of material as they arose from the discussion with the teacher and the questionnaires of the teacher and students.

Computer programs: The programs were used as illustrative materials and as discovery. The students worked in pairs on a microcomputer in their classroom while the rest of the class performed other tasks. They seemed to enjoy them. The time spent on them was about 15-30 minutes. The teacher found the computer programs 'excellent', much improved from their first version, what she would like to prepare by herself but could not because of lack of time. She found them the most successful in the classroom.

Consolidation and practice exercises: The teacher found them very useful, especially for the low attainers. Some small blunders in some questions were identified.

Practical activities: The students worked in groups and they discussed the outcomes of each activity. Despite the fact that it was something new for them, they seemed to enjoy it. The teacher used three activities in the classroom. One, the radian measure, was done as part of the lesson under her direction while the others were done by the students themselves. She explained that the main reason for using just a few activities was the lack of time. She believed that "practical activities will get more successful as students - and lecturer - get accustomed to that approach". This kind of work also helped her to discover unexpected abilities in her students. For example, a low attainer with language problems got on very well in a practical activity and showed understanding which she had not appreciated before. The results from the students' and teacher's questionnaires on the practical activities are summarised below.

Practical activity 16 (quadrilaterals): 2 students.

Time spent: 30 to 45 minutes.

Students' responses: One found it interesting, while the other did not.

Teacher's responses: It was something new for the students and it took too long.

Practical activity 18 (corridors): 4 students.

Time spent: 20 to 40 minutes.

Students' responses: Most found it interesting. Some had problems with some instructions in the record sheet.

Teacher's responses: She liked it because it showed a practical application.

Practical activity 19 (radian measure)

Teacher's responses: Very worthwhile and effective. The students liked it.

Posters: The students noticed them after encouragement but there was not any particular response. The poster about Errors was used in connection with a problem solving activity.

From the evidence of the short visit and the discussion with the teacher, the students worked in groups when they were doing practical or computer work, for other activities they worked individually. The teacher seemed to be willing to change her approaches to teaching, although, as she said, she still had some difficulties. She had good relationships with the students and could act sometimes as a counsellor, because the class was a small size (9 students). The main teaching approaches were lecturing and discussion. The students were very cooperative. She regarded a timetable of three hours continuously teaching as too much for the students. In addition to the methods of assessment suggested in the guidelines she assessed the course by problem solving assignments.

Results from the final questionnaires: Nine students were in the class and seven completed the questionnaires. All students liked the course and they thought it useful for their future. Most noticed an improvement in their ability and they liked mathematics more after the course. All had friendly relationships with the teacher and the other students. How they found the various topics of the module, which materials they used and how they found them are summarised in Tables 3.14 and 3.15.

Table 3.14

	Topics				
	Errors	Linear funct.	Quadrat- ic funct.	Trigon- ometry	Vectors
Very difficult	0	0	0	0	0
Difficult	2	1	0	3	0
Average	2	4	3	1	4
Easy	3	2	3	3	2
Very easy	0	0	1	0	1

Table 3.15

	Computer programs	Practical activities	Consolidation exercises	Posters	Notes
Number of users *	7	7	7	7	7
Very interesting	1	0	0	0	0
Interesting	4	4	3	4	5
Not interesting	2	3	3	3	1
Very helpful	1	0	0	0	6
Helpful	6	6	5	4	0
Not helpful	0	1	1	3	0

* The discrepancies between those who used the materials and those who made comments on them are because some students did not complete the questions carefully. From the tables it seems that most students found the topics of average difficulty and the materials interesting and helpful. All students had 0-grade mathematics and one had also Higher grade. One suggested using more practical activities.

From the teacher's questionnaire it can be seen that the teacher also used notes, handouts, worksheets and programs from the college's stock of software. The students found the course worthwhile and interesting. She would like to use the computer programs, practical activities, posters, in the future. She would prefer a more detailed descriptor, for content, and clearer assessment procedures.

Results from interviews: Seven students were interviewed. The interviews gave some information about the students background. They were full-time students and they had to do the module as part of their course. All the class did not follow the same course, some students were doing data processing while others, computers. Apart from one student, nobody had previous experience of college life and they preferred the life in college as being more informal than in school. The interviews also showed some of the students' attitudes towards the mathematics course and its individual elements. Most liked the course and thought it useful for their future. Some students did not like mathematics but they had to do it. Most liked working in groups. Most found trigonometry difficult and some had problems in error algebra and factorization of expressions. Most enjoyed the practical work. One said "It is not boring. You have fun and at the same time you learn". One did not like it because "It gets on your nerves". All liked the consolidation exercises as they revised things which they had forgotten. Most preferred the computer programs. Everybody had noticed the posters but did not pay any attention to them. They found the assessment worksheets of average difficulty and sometimes they completed them as homework.

Results from school

The materials were used by a class of eighteen students. Four of them left school during the study, having reached school-leaving age by Christmas. The students had taken Standard grade or O-grade examinations and they were not following the Higher Mathematics course but instead they were taking three modules ($M_2|A_1$, $M_2|A_2$, $M_2|A_3$) over two years. At the end of the second year they could sit the Higher examinations. Most students had obtained poor grades in the Standard or O-grade examinations in mathematics, although the level of ability varied a lot. This system of using modules instead of the Higher course was adopted by the school to give more opportunities to students who would most probably fail the Higher mathematics examinations. The timetable was one period mathematics per day and the module $M_2|A_1$ lasted approximately three to four months with a break in December to enable the pupils to prepare for their preliminary Higher examinations.

The teacher's initial contact with the materials was before the course started. The author had three meetings with the teacher. Two of the meetings took place in the school, observing the pupils in the classroom using the materials.

Some important points which emerged from the meetings, discussions with teacher and pupils, questionnaires, written reports on investigations are summarised below for each set of materials. The author, in the short visits, saw being used only the practical activities and the consolidation and practice exercises.

Computer programs: The students used the programs in the classroom. A small group was working on the computer while the rest of the class were involved in another activity. The programs were used as illustration and investigation materials. The students found

them interesting on the whole, although many did not like the activity very much. They did not find any particular difficulty. As the teacher commented, the use of the computer room was limited but she could always have a computer for use in the classroom.

Videotaped expositions: Some of the programmes were used by the class as a whole in the classroom. The students could not concentrate for a long period on a programme so the teacher had to stop the tape and explain the main points of it. Discussion and consolidation after each programme were taking place. The students enjoyed them if used occasionally but they had a problem with the language used in the programmes because the notation used sometimes was different from that used in school.

Consolidation and practice exercises: The students used the exercises mainly for ten minutes at the beginning of each period. The teacher and the students found them useful. The teacher observed the students' work and through it discovered difficulties for example in the order of operation, indices, etc.

Practical activities: The students worked in small groups and there was a general discussion afterwards. The students had difficulties in appreciating the mathematics involved in the activities. This was the case particularly at the beginning of using this approach. Some students could not find any reason for using the activities, while they could do the 'formal mathematics' instead. After the first session of practical work the students responded more enthusiastically. The teacher found the practical work useful in helping students to appreciate the use of mathematics in different contexts, but sometimes the approach was too time consuming. The results from the students' and teacher's questionnaires on the practical activities are summarised below.

Practical activity 1 (plan of classroom): 5 students.

Time spent: 25 to 50 minutes.

Students' responses: Interesting.

Teacher's responses: Worthwhile. The students liked it.

Practical activity 2 (whisky bottle): 5 students.

Time spent: 15 to 60 minutes.

Students' responses: Most found it interesting. Some could not find reasons for using it as "It has no link to actual Higher work".

Teacher's responses: "Very worthwhile activity both for error and graphing". Students had some difficulties in drawing the graph.

Practical activity 3 (toilet roll): 14 students.

Time spent: 15 to 50 minutes.

Students' responses: Most did not like it. Some difficulties in using the micrometer. Some found the task boring and not worthwhile. They would prefer it more 'theoretical'.

Teacher's responses: Class did not realise the significance of the exercise. Some difficulties in using the micrometer.

Practical activity 5 (pipes): 2 students.

Time spent: 60 minutes.

Students' responses: Interesting.

Teacher's responses: Students liked it. Worthwhile.

Practical activity 14 (parabolic table): 2 students.

Time spent: 30 minutes.

Students' responses: Equipment did not work.

Teacher's responses: Frustrating because of difficulty in getting the ball-bearing to rebound from the board.

Practical activity 15 (minimum of quadratic function):
11 students.

Time spent: 50 to 60 minutes.

Students' responses: Most did not like it. Some problems with the instructions of description sheet.

Teacher's responses: Students did not like it too much. Too time-consuming. Instructions ambiguous. It would be interesting if it could be shortened.

Practical activity 16 (quadrilaterals): 6 students.

Time spent: 35 to 60 minutes.

Students' responses: Most found it interesting. One suggested clearer instructions.

Teacher's responses: Worthwhile. Students liked it.

Practical activity 17 (cylinders): 7 students.

Time spent: 30 to 60 minutes.

Students' responses: Most found it interesting. Some equipment was not suitable (eg. pair of scissors).

Teacher's responses: Worthwhile. Students liked it. More posts could be used of different shapes.

Practical activity 19 (radian measure): 3 students.

Time spent: 30 minutes.

Students' responses: All found interesting or very interesting.

Teacher's responses: Worthwhile. The students liked it.

Tape-booklet sequences: The sequences were used by the students at home as a tape recorder was not available in the college. Most students liked them. They used them mainly for revision. Some students who were absent used them to catch up with the rest of the class. Most students found them very easy to follow although some found them repetitive in parts because they already knew the relevant skills. Most found them helpful.

Posters: The students did not notice them by themselves. There was discussion only for a short time. The teacher did not notice any particular reaction to them. She found the posters too small for class discussion and they had to be used in groups.

Investigations: Three investigations were used in the module: the best way of calculating the square root of two, the design of a milk carton, and a series of equilateral triangles (parabola). The students did these at home and they produced a written report. They found them difficult and most did not enjoy them. The teacher found this approach useful. It was quite difficult to help the students suitably and assess their work. She also remarked that the students were not used to this approach and they preferred 'traditional' teaching. From the investigations' reports emerged some abilities, difficulties and misunderstandings of the students. They revealed the students' understanding in some mathematical processes. For example, in the calculation of the square root of two a lot of pupils had not understood how to use the iteration methods. They had not appreciated the meaning of "approximation to seven decimal places", (they gave 1.4166667 as an answer because it had 7 decimal places). Difficulties also appeared in the 'decision making' process. For example, they did not explain clearly what criteria they were using in deciding what was the "best" method for $\sqrt{2}$ or what was the "best" milk carton. They also had difficulties in creating and analysing a mathematical model. In the investigation about the series of equilateral triangles, the students showed a lack of ability to generalise from patterns. As expected, the reports also revealed that students used different methods of approach depending on their previous experience. For example, for the calculation of the square root of two, some tackled the problem graphically

while others tried to interpret some numerical data. In addition, some lack of mathematical skills also emerged. For example, difficulties in indices were diagnosed. The students' ways of communicating their methods and ideas could also be examined and improved. There was also some indication that all students can perform well in the kind of activity regardless of their abilities in the formal mathematical procedures. For example, a student who twice gained the prize in the investigations competition was regarded by the teacher as lazy and not particularly able.

It appeared from the short visits that the teacher had a friendly relationship with the students. Normally they worked individually, for practical work and for the computer work they worked in groups.

Results from the final questionnaires: Thirteen students completed the questionnaires. Almost everybody liked the course either because of its being enjoyable or because of the use of continuous assessment, or because of its covering a lot of topics. Some did not like the investigations. All found the course helpful. Most noticed an improvement in their mathematical ability and they liked mathematics the same as before the module. They found their relationships with the teacher and other students very friendly. How they found the various topics of the module, what materials they used, and how they found them is summarised in Tables 3.16 and 3.17.

Table 3.16

	Topics				
	Errors	Linear funct.	Quadrat- ic funct.	Trigonom- etry	Vectors
Very difficult	0	2	3	0	0
Difficult	1	5	3	3	3
Average	7	4	4	4	6
Easy	4	1	1	4	2
Very easy	1	1	2	2	2

Table 3.17

	Computer programs	Practical activities	Consolidation exercises	Posters	Videotapes	Investigations	Tape Booklets	Books	Notes
Number of users*	13	13	9	-	12	13	11	10	3
Very interesting	0	1	0	-	0	1	2	1	0
Interesting	7	10	6	-	5	7	7	6	3
Not interesting	6	2	3	-	7	5	2	3	0
Very helpful	1	1	2	-	2	1	4	2	0
Helpful	5	8	5	-	8	7	6	7	3
Not helpful	7	4	2	-	2	5	1	1	0

* The discrepancies between those who used the materials and those who made comments on them are because some students did not complete the questions carefully.

From the tables it seems that the students found the topics of average difficulty, although many found the linear and quadratic functions particularly hard. None wrote that posters were used. They found most materials interesting and helpful apart from the computer programs, which most students did not find helpful, and the video programmes, which most did not find interesting. The book which was used was Modern Mathematics for Schools (Blackie-Chambers). Almost everybody had Standard grade mathematics (grades from 2 to 4, but mainly 3), apart from one who had O-grade arithmetic. Some students wanted better investigations, others more notes and clearer study books for the course, more self-help materials, more practical work. One commented "a greater emphasis on practical applications of mathematics". One had difficulty in keeping track of what he was doing because of the use of different resources. They also expressed anxiety about their future following the modular pattern. Some found difficulties in using some materials. One student wrote "I did not find the videotapes or computer programs much help because they tended to just pass you by. Unless you are actually doing the examples then it just becomes meaningless".

From the teacher's questionnaire it seems that the teacher had good and relaxed relationship with the students. The students were cooperative but sometimes the school restricted the cooperation with each other. Her main approaches in helping the students were by lecturing, discussions and individual help. On the average in a 4 hours' session, she spent most of the time in exposition including consolidation by the students (3 hours), half-an-hour on the computer programs and half-an-hour in practical work. She found the computer programs, the practical activities and the video programmes most successful in the classroom. She felt that the students found the course enjoyable,

worthwhile and "let them see that maths need not be all done at a desk". She assessed the course by worksheets, end of module tests and investigations' reports. She would like to use all the materials in the future, particularly the computer programs. She would prefer some different investigations. She regarded the materials as more than adequate for the course. She commented on the dissemination of material and ideas "regions must run courses, advisors must take an interest, or development officers must be appointed, to go into schools". Seminars, inservice training, resource libraries where information and ideas could be exchanged were regarded as essential. More standardised assessment procedures would be helpful. She added that she liked the materials and "without your help and support the course would have been very boring and dull".

Results from interviews: From discussions with the students, it emerged that some students found some materials very helpful, while some others could not see their effectiveness in learning mathematics. For example, some found the use of investigations and the writing up of their work on them not as useful as the 'traditional' mathematics. Most had not noticed the posters. They liked the consolidation and practice exercises and the tape booklets. Most found the course difficult in some parts.

From the discussion with the teacher some further points arose. She remarked on the limited time available for covering the content of the module, her inexperience in helping students with investigations and her tendency to think about the Higher Examinations (in terms of content).

CHAPTER FOUR

CONCLUSIONS

The successful development of the learning resources produced in this study demonstrated that it is possible to create the learning environment suggested in the guidelines on teaching/learning approaches.

The production of the materials required much work, use of various sources of ideas, discussions with teachers, students and people involved in the development of the module. In designing the materials, an important consideration was the presentation of the mathematics within contexts relevant to the students' interests. The materials attempted to give to all the students opportunities for discovering by themselves mathematical concepts and ideas and for developing their mathematical skills. They also aimed to encourage the students to become independent learners by managing their own learning.

The time spent producing the materials is an important factor. The computer programs were the most time consuming materials. This was partly because of the limited programming experience of the author. The practical activities required a considerable amount of time. In addition to the difficulty of finding appropriate ideas, the production of the equipment and the writing of the description sheets was sometimes time consuming. Some more structured materials, for example, tape-booklet sequences or the consolidation and practice exercises, needed a careful selection of examples and exercises so as to help students to consolidate various skills. They also required time for typing. An estimate of the time spent on the production

of the materials in this study is shown in figure 4.1. Of course the time spent depends on many factors,

Figure 4.1

Materials	Production time
Computer programs	6-7 months
Videotaped expositions	1-2 months
Consolidation exercises	1-2 months
Practical activities	3-4 months
Tape-booklet sequences	2-3 months
Posters	2-3 months
Investigations	1-2 months

for example, programming expertise, artistic ability, experience of producing self-learning materials, teaching experience, etc. It is clear from this analysis that it is unreasonable to expect the class teacher to produce all these materials. Indeed, it is unrealistic to do so. As well as being responsible for developing teaching/learning materials for the course, the teacher has a major responsibility for its assessment which is a time-consuming activity. It is more realistic to expect the teacher to select and adapt existing materials to suit the needs of his/her students. The initial task then is the selection of materials for the course and the options are somewhat circumscribed by the financial resource available. Figure 4.2 shows the estimated cost of the production of the materials for a class of twenty students. The highest cost is that of the video taped expositions and of the tape/booklet sequences. The cost of the investigations was the cheapest. It is very difficult to decide which materials are the most cost-effective because their effectiveness

Figure 4.2

Materials	Estimated costs	Comments
Computer programs (in 2 discs)	£5.00 (one set of discs)	Including worksheets and teaching guide.
Videotaped expositions	£120.00 (one set)	Including teaching guide.
Consolidation exercises	£20.00	Including teaching guide.
Practical activities	£3.50 + cost of equip- ment	Including teaching guide and description and record sheets. On average the cost of equipment is about one to two pounds per item.
Tape/ booklets	£37.00 (one set)	Including teaching guide.
Posters	£8.00 (one set)	Including teaching guide.
Investigations	£1.80	Including cost of duplicating 4 investigation statements to students and teaching guide.

depends on the needs, interests, aptitudes of the individuals. Indeed the question of cost-effectiveness is inappropriate.

The information in Figures 4.1 and 4.2 is provided to give information to teachers about the production time and costs of the materials. They are not intended to provide a basis for judgements between materials.

Teachers can prepare some materials for their classrooms with suitable support from their colleges, for example, less teaching hours, provision of resources. They could also be seconded for developmental work; development teams can be created and the sharing of ideas and materials can be encouraged.

From the evaluation study it seems clear that all the materials created positive attitudes towards mathematics for most students. They created a different atmosphere in the classroom, caused discussion, co-operation and provided a variety of mathematical experience. The success of the materials will increase as teachers and students become familiar with these new approaches. In addition, factors such as the college organisation and classroom environment play an important role in the effective use of these resources. A mathematics workshop with the appropriate equipment, for example, microcomputers, video equipment, tape recorders, books, equipment for practical activities, could help teachers to organise the resources and create an appropriate learning environment.

The main problem identified by the evaluation was the shortness of the time period of the module. Because of this and the vagueness of the description of content in the module descriptor, teachers gave more emphasis to the verbal exposition, neglecting other approaches. This problem can be overcome by the use of self-learning materials. The teacher by himself cannot meet all the needs of the students in the time allocated.

For example, tape-booklet sequences, consolidation and practice exercises can be used by the students in their own time without the teacher's involvement. Investigations can also be undertaken outside class hours, but the appropriate guidance and discussion is essential before and after the students tackle the problems. Students have to be given opportunities to work by themselves in and outside the classroom if they are to become independent learners.

In addition to the approaches suggested in the study, the use of textbooks, reference books, notes, worksheets, is essential for facilitating students' learning. Well equipped libraries in each institution will help students in their learning process.

An important element of the module is the formative and summative assessment of the students' work in the different activities. This has not been formally investigated in this study but both observation and the questionnaires revealed that it is a key issue for the class teacher.

The use of the learning approaches suggested in the guidelines will be successful as teachers and students become accustomed to the new approaches. Suitable support is needed to hasten that process. This support can come from the institution and the educational authorities. The appropriate environment in the institution will engender the cooperation of different departments which will help teachers and students to realise the new approaches. The availability of equipment is vital. Seminars, inservice training, sharing of ideas and materials between institutions, counselling by advisors and development officers will help teachers to meet the demands of their new role. Their ability to fulfil their new role vitally affects the success of the students who need support and

guidance in their learning.

The materials described in this study have assisted teachers in their attempts to meet the new demands of their work. The ideas arising from these materials offer something new in the learning of mathematics and give opportunities for further developments in mathematical education.

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APPENDICES

Volume 1

Appendix A Descriptor of Mathematics Module M_2/A_1

Appendix B Initial phase of evaluation

Evaluation of computer programs (teacher's
and student's questionnaires, short tests).

Evaluation of consolidation and practice
exercises (teacher's and student's
questionnaires)

Appendix C Final phase of evaluation

Evaluation of practical activities (teacher's
and student's questionnaires)

Teacher's instructions for the investigations
competition

Final questionnaires (teacher's and student's
questionnaires)

Volume 2

Appendix D Computer Programs

Teaching guide

Two discs (40 tracks)

Worksheets

Documentation of the programs

Appendix E Videotaped expositions

Teaching guide

Script for the expositions on trigonometry

Appendix F Consolidation and practice exercises

Teaching guide

Booklets

Appendix G Practical activities

Teaching guide

Description and record sheets

Appendix H Tape/booklet sequences

Teaching guide

Booklets

Audio-tapes

Appendix I Posters

Teaching guide

Posters

Appendix J Investigations

Teaching guide

Description sheets

APPENDIX A
Module Descriptor

SCOTTISH VOCATIONAL EDUCATION COUNCIL

22 Great King Street
 EDINBURGH EH3 6QH
 031-557 4555

38 Queen Street
 GLASGOW G1 3DY
 041-248 7900

NATIONAL CERTIFICATE MODULE DESCRIPTOR

Ref No.	61058	Session 1986-87
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Title	MATHEMATICS: ANALYSIS/ALGEBRA 1	M2/A1
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Type and Purpose	A <u>general</u> module which is designed to present post-Standard Grade mathematical concepts within the context of applications. It should enable the student to consolidate previous ideas and concepts and to acquire the knowledge and skills from which the study of the subject can be developed. Refer to Appendix 1 for more guidance.
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Preferred Entry Level	0-Grade Mathematics band C, Standard Grade in Mathematics at 3, 01057 Mathematics: Grade 3 or equivalent.
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Learning Outcomes	The student should: <ol style="list-style-type: none"> 1. understand the role of accuracy of number in practical applications and calculate the error bounds for a numerical result; 2. understand the properties of linear, quadratic and circular relationships; 3. distinguish between scalars and vectors and use vector algebra in simple applications and apply the understanding in a practical context; 4. integrate mathematical knowledge and skills in a problem solving context; 5. maintain a workfile competently.
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Content/Context	<ol style="list-style-type: none"> 1. express and use numbers to a specified number of significant figures or decimal places. Express and use numbers in scientific notation. Calculate absolute and relative errors and apply rules for error bounds. Apply the above to practical measurements and calculations. 2. read values from graphs and interpret gradient. Draw linear graphs from supplied data. Express linear graphs as an algebraic function and establish relationships between
-----------------	---

two variables. Use the above techniques to solve practical problems. Solve linear equations taking account of appropriate error bounds. Determine specific coordinates and aspects of a quadratic graph. Solve quadratic equations graphically and analytically. Use the trapezoidal rule to determine approximate areas between a coordinate axis and a quadratic curve. Appreciate rate of change and average rate of change. Apply the above to practical problems including velocity, acceleration and results of experiments. Use trigonometric ratios to solve problems involving right-angled triangles; including appropriate error bounds. Graph trigonometric ratios. Measure angles in degrees and radians. Solve problems requiring sine, cosine and area formulae. Use trigonometry to solve practical problems.

- 3. distinguish between vector and scalar quantities. Use 3-dimensional vector algebra to model and solve problems. Draw and use 2-dimensional vector diagrams to solve problems. Use vector components, unit vectors and position vectors, as appropriate, to solve practical problems.
- 4. complete problems/investigations involving the integration of the content of Learning Outcomes 1, 2 and 3.
- 5. the workfile should form a complete record of the student's work-throughout the module.

Suggested Learning and Teaching Approaches

Emphasis should be placed on graphical work to reinforce basic concepts.

Computer access is assumed. Computer packages should be used where appropriate and advantage taken of any programming skills possessed by the student.

Suitable computer packages are available from the SMDP Supported Software Library.

Group problem solving and practical investigation are essential parts of the module.

Self-help remediation material should be used to reinforce skills where necessary. For example tape/booklets on topics such as the parabola, sectors and errors are available.

Much time can be saved by the use of pre-prepared worksheets.

Assessment
Procedures

All Learning Outcomes must be validly assessed.

The student must be informed of the tasks which contribute to summative assessment. Any unsatisfactory aspects of performance should, if possible, be discussed with the student as and when they arise.

Acceptable performance in the module will be satisfactory achievement of the performance criteria specified for each Learning Outcome.

Where cutting scores are stated these are intended to be for guidance. The precise cutting score for a test will depend on the difficulty of the test.

The following abbreviations are used below:

LO Learning Outcome
IA Instrument of Assessment
PC Performance Criteria

L01 IA Diagnostic worksheets. The worksheets
2&3 should form part of the regular teaching programme and should be issued at intervals throughout the module. The style of worksheets should be varied. Some may be highly structured showing students how to set out work systematically, others may merely record the results of work written on separate pages or performance on a calculator or computer.

PC Normally at least 75% of calculations, mathematical manipulations and presentations or interpretations of information should be carried out correctly.

L01 IA Short answer questions could, for
2&3 convenience, be combined together within a single test paper which would be of approximately one or one and a half hours duration. It should be possible to identify the questions which relate to individual Learning Outcomes.

PC Normally at least 75% of questions associated with EACH Learning Outcome should be completed correctly. In cases where a student fails to meet the criteria for a particular Learning Outcome, the student's workfile should be consulted. If the examiner is satisfied there is sufficient evidence that the student has performed satisfactorily the student may be credited with having achieved the Learning Outcome.

Continuation of Module 61058

Session 1986-87

L04 IA Extended answer questions. The extended answer question test should be set towards the end of the module. The questions should test the ability of the student to draw together various mathematical ideas and techniques developed in the module and use them to solve simple problems, many of which will be expressed within a practical context. Calculators may be used if required.

PC The student should perform to a standard acceptable to the examiner and should demonstrate an ability to:

- (a) interpret a problem;
- (b) select a strategy to solve the problem;
- (c) draw together various mathematical ideas and techniques developed in the module;
- (d) obtain satisfactory solution;
- (e) communicate accurately and logically.

*L05 IA Student workfile. The student workfile provides an opportunity to encourage good work habits and develop sound study skills in the student. The lecturer should ascertain periodically throughout the module that each student is maintaining his/her workfile adequately. The workfile should contain (as appropriate) the student's notes, class handouts, completed worksheets, exercises, assignments, reports of investigations, reports of projects, log books of computer activities and a summary of the important details of the module, for later revision purposes.

- PC
- (a) at least 75% of each worksheet or exercise should be completed correctly;
 - (b) writing should be clear;
 - (c) numerical work should be accurate;
 - (d) graphical work should be carefully drawn and annotated;
 - (e) computer programs (where used) and results should be carefully documented;
 - (f) there should be references for the sources of the exercise and notes.

APPENDIX B

Evaluation of computer programs

Teacher's questionnaire

Programme:

College: _____

Lecturer: _____

Date: _____

No. of students in class

1. Did the students like the programme?
2. Were there any parts of the programme where students found difficulties?
3. How long was spent using the programme and completing the worksheet?
4. Did all the students use the programme?
5. Put a tick (✓) in your option below:

Did they find the worksheet

very easy	<input type="checkbox"/>
easy	<input type="checkbox"/>
OK	<input type="checkbox"/>
difficult	<input type="checkbox"/>
very difficult	<input type="checkbox"/>

6./

6. Are there other topics which you could teach with the use of the computer in this week's work?
7. Choose one or more options putting a tick (✓) in the box chosen.
How was the programme used?
- | | |
|------------------------------|--------------------------|
| a) illustrative material | <input type="checkbox"/> |
| b) reinforcement material | <input type="checkbox"/> |
| c) by whole class as a group | <input type="checkbox"/> |
| d) by individuals | <input type="checkbox"/> |
8. Please list the materials which you used in this week's work,
e.g. textbook, computer ...
9. Please describe how you assess this part of the course,
(e.g. homework, class test). Please indicate time taken
for this by the student.
10. Please indicate which activities you used for this part of
the course:
- | | | | |
|------------------------------------|--|-----|----|
| (a) exposition of the topic by you | <table border="1"><tr><td>yes</td><td>no</td></tr></table> | yes | no |
| yes | no | | |
| (b) discussion with students | <table border="1"><tr><td>yes</td><td>no</td></tr></table> | yes | no |
| yes | no | | |
| (c) discussion between students | <table border="1"><tr><td>yes</td><td>no</td></tr></table> | yes | no |
| yes | no | | |
| (d) practical work | <table border="1"><tr><td>yes</td><td>no</td></tr></table> | yes | no |
| yes | no | | |
| (e) consolidation and practice | <table border="1"><tr><td>yes</td><td>no</td></tr></table> | yes | no |
| yes | no | | |
| (f) problem solving | <table border="1"><tr><td>yes</td><td>no</td></tr></table> | yes | no |
| yes | no | | |
| (g) investigational work | <table border="1"><tr><td>yes</td><td>no</td></tr></table> | yes | no |
| yes | no | | |

Thank you very much for helping me with this development work and for answering these questions.

Please write any comments you may have about this program on this sheet, e.g. any future improvements that could be made to the program, how you would use the program with future classes, etc.

Student's questionnaire

Programme Appraisal (Please return this part to Miss D. Potari,
for checking)

Name: _____

Date: _____

E R R O R S

(1) Did you find the programme

very interesting ☐

interesting ☐

not interesting ☐
at all

Choose one option putting a tick (✓) in the box.

(2) Did you find the worksheet

very easy ☐

easy ☐

difficult ☐

very difficult ☐

Choose one option putting a tick (✓) in the box.

(3) Did you find some difficulties?

Yes ☐

No ☐

Choose one option putting a tick (✓) in the box.

If 'yes', please specify shortly what difficulties you had.

(4) Can you suggest any improvement in the programme which you think
that will help you?

(5) Do you have any comments about this part of the course?

Short tests

Errors

1. Complete the table below where a is a correctly rounded number and ϵa its absolute error bound.

a	ϵa
3.567	
-0.5	
4.500	
2.0	
0.06	

2. With $a =$ and $b =$ the programme gives

$(a/b)_{\min} =$

and

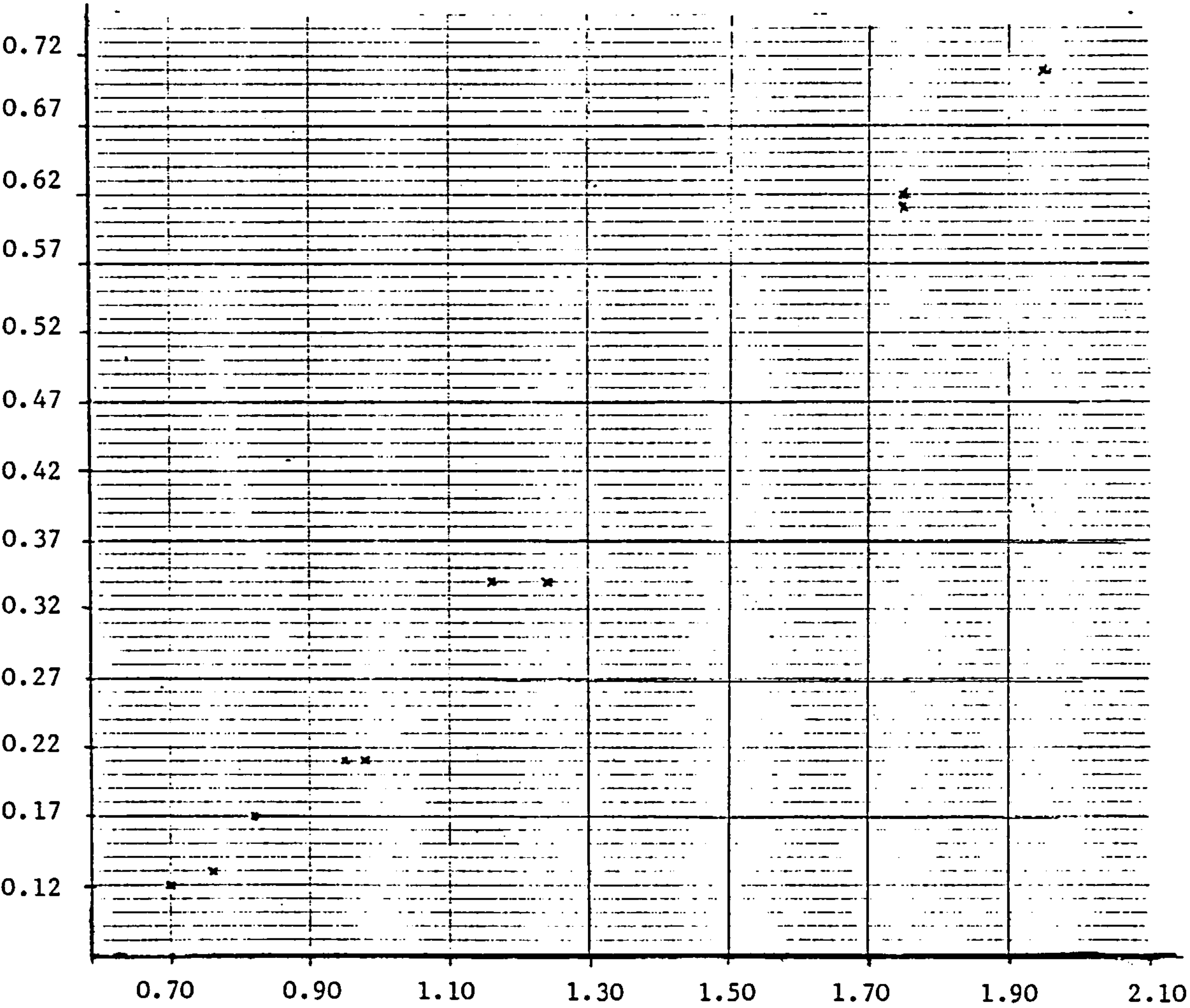
$(a/b)_{\max} =$.

What is a sensible value for a/b ? Write your answer
in the next box

Line

The hardness x of a piece of plastic is related to the amount y of a certain substance which is in the plastic. Data were collected and are given in the table below.

x	0.70	0.98	1.16	1.75	0.76	0.82	0.95	1.24	1.75	1.95
y	0.12	0.21	0.34	0.61	0.13	0.17	0.21	0.34	0.62	0.71

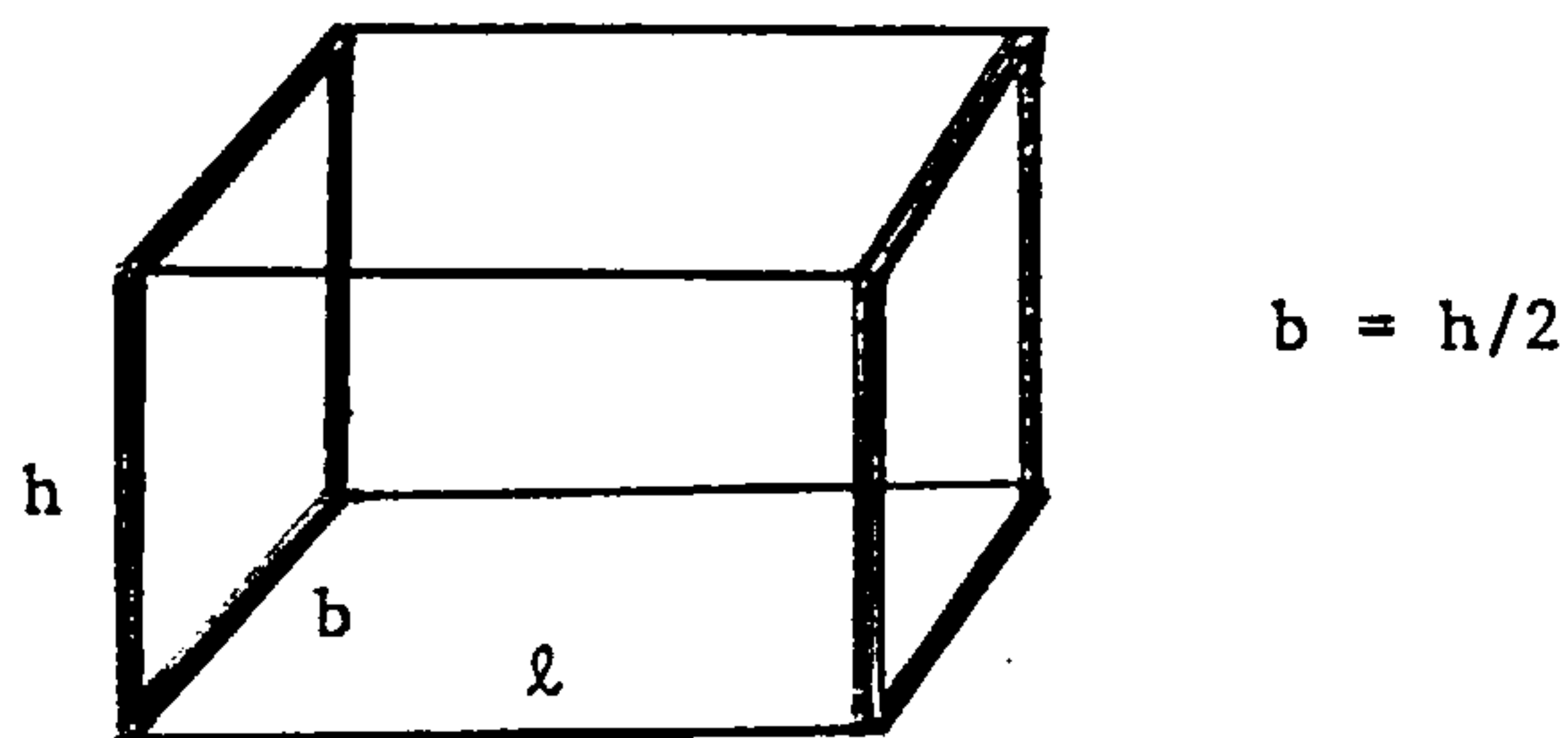


- a) Use the programme to find $y = \boxed{} x + \boxed{}$ and draw the line on the graph paper above.
- b) If $x = 1.10$ what is the corresponding value of y together with its error interval?

Minimum

We want to make a box whose breadth b is half of the height h and of volume $V = 500$ ml.

The two sides of the box are three times as thick as the other sides. The length of the box is ℓ . We want to find the dimensions of the box where the cost is as small as possible.



The cost is proportional to the area A of the box so A must be as small as possible.

The volume V as a function of h and ℓ is

$$V = \boxed{(h^2 \ell) / 2}$$

and the area A as a function of h and ℓ is

$$A = \boxed{3h^2 + 3h\ell}$$

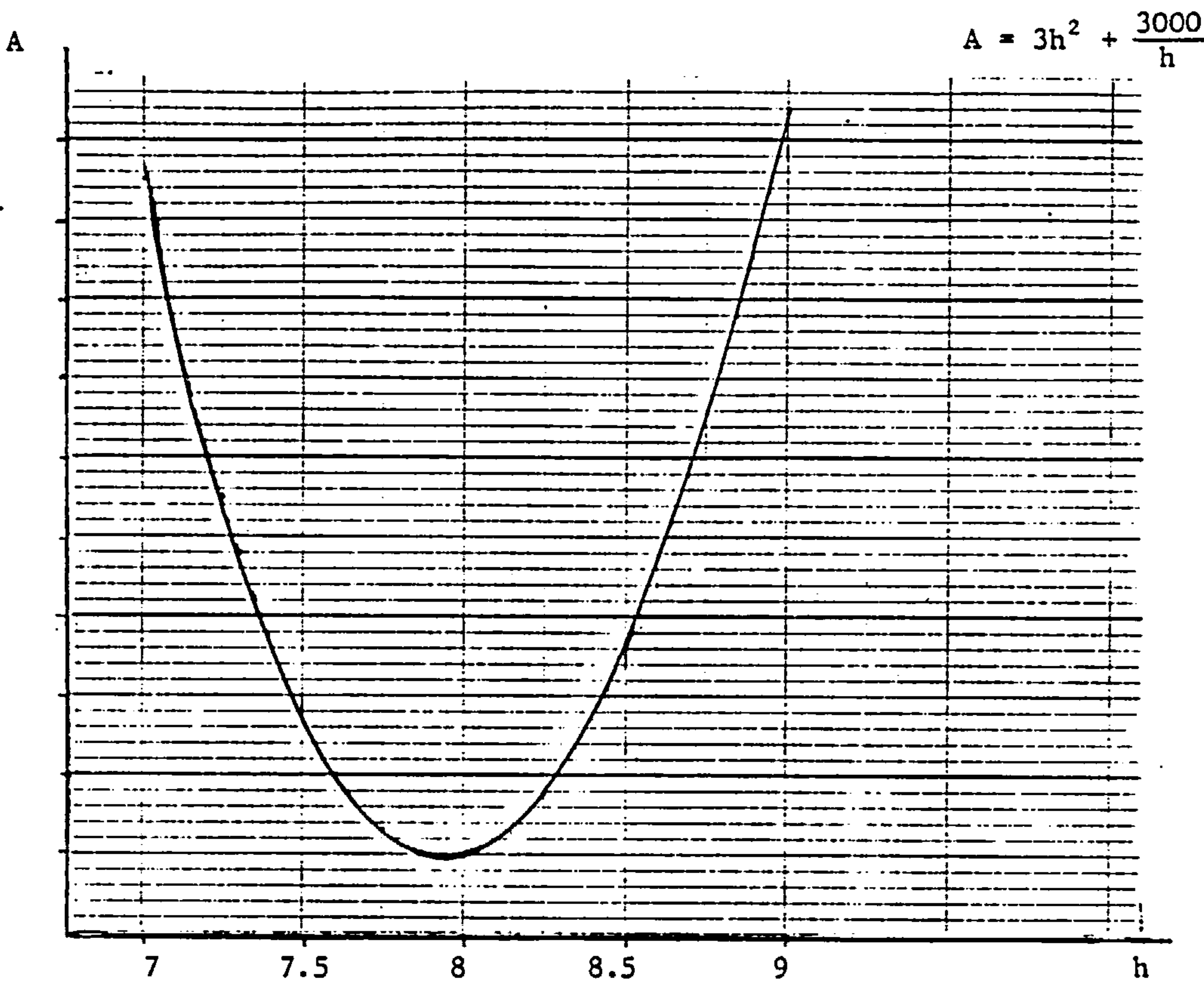
When the volume V is 500 ml, the length ℓ as a function of h is

$$\ell = \boxed{1000/h^2}$$

The area A as a function of the height h is

$$A = \boxed{3h^2 + 3000/h}$$

The graph of the area A as a function of h is drawn below.



Estimate from the graph the value of h for which the area A has its minimum value (in two decimal places)

$h =$

$A =$

Use a calculator to find, if possible, a better estimate of the optimal value of h . Give below your improved estimates and the corresponding values of A :

h (to 2 decimal places)									
A									

Now give the optimal dimensions of the box:

$h =$ mm.

$l =$ mm.

$b =$ mm.

Quadratic Function

- 1) The solution of the equation $ax^2 + bx + c = 0$, graphically, is the values of x where the function $f(x) = ax^2 + bx + c$ cuts the x -axis.

If the graph cuts the x -axis in two points the equation has two roots.

If the graph touches the x -axis the equation has one root.

If the graph doesn't cut the x -axis the equation doesn't have any root.

Using the programme, complete the table below. Put 'n.a.' if the equation doesn't have roots.

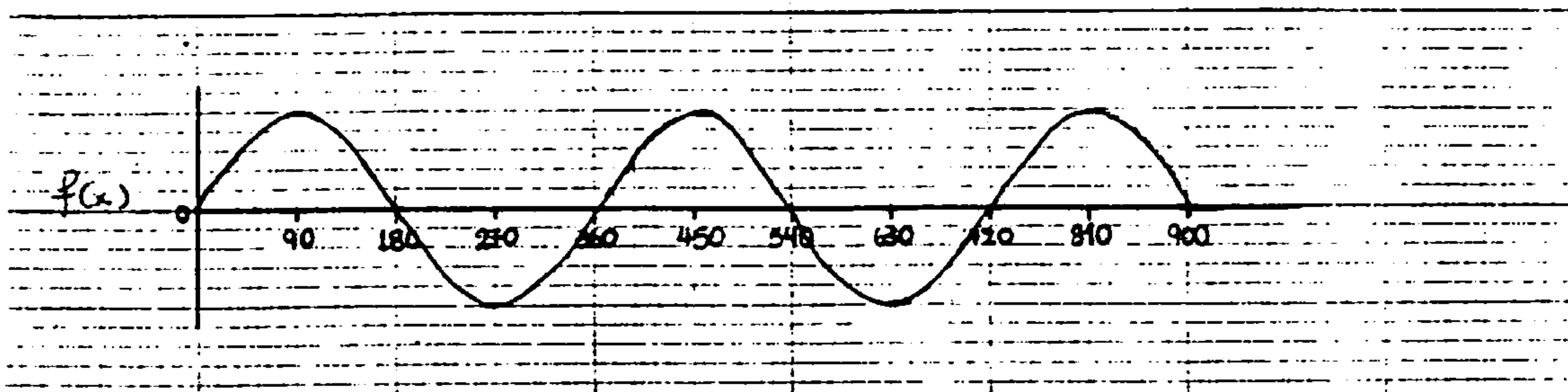
equation	number of roots	roots
$x^2 - 6x + 9 = 0$		
$-x^2 - x + 6 = 0$		
$4x^2 + 9x + 10 = 0$		

- 2) Using the programme, please complete the table below.

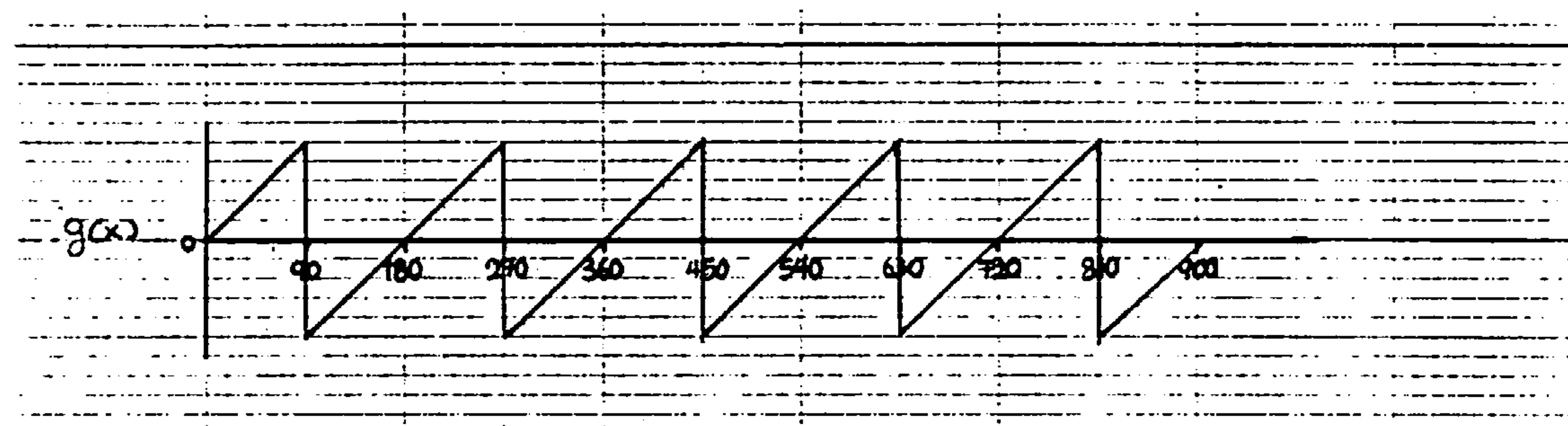
$f(x)$	Shape of curve	Turning point	Nature of turning point
$x^2 + 4x + 6$			
$-2x^2 + 8x - 4$			
$x^2 + 6x + 9$			

Cosine-Sine curves

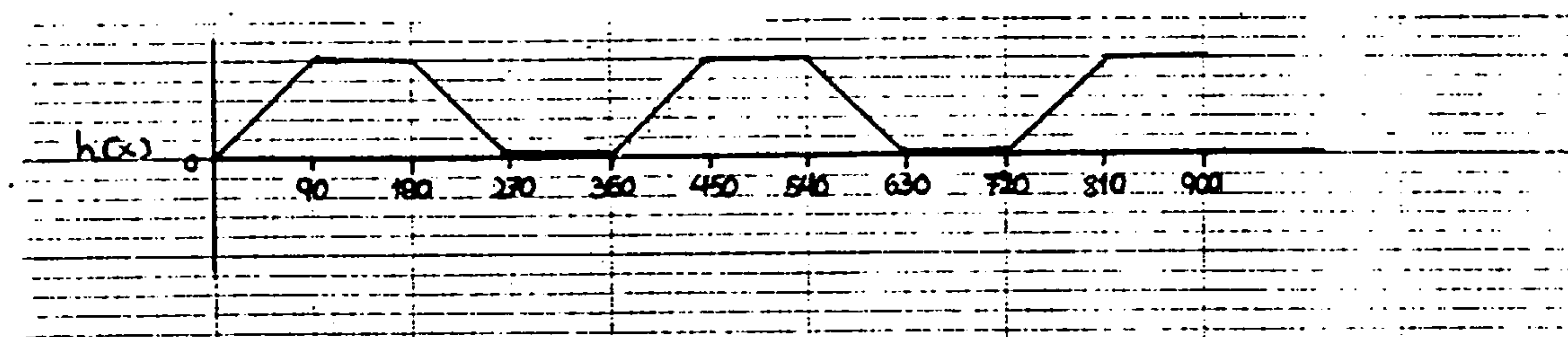
1) What is the minimum period of the functions, below



The minimum period of the function $f(x)$ is degrees.



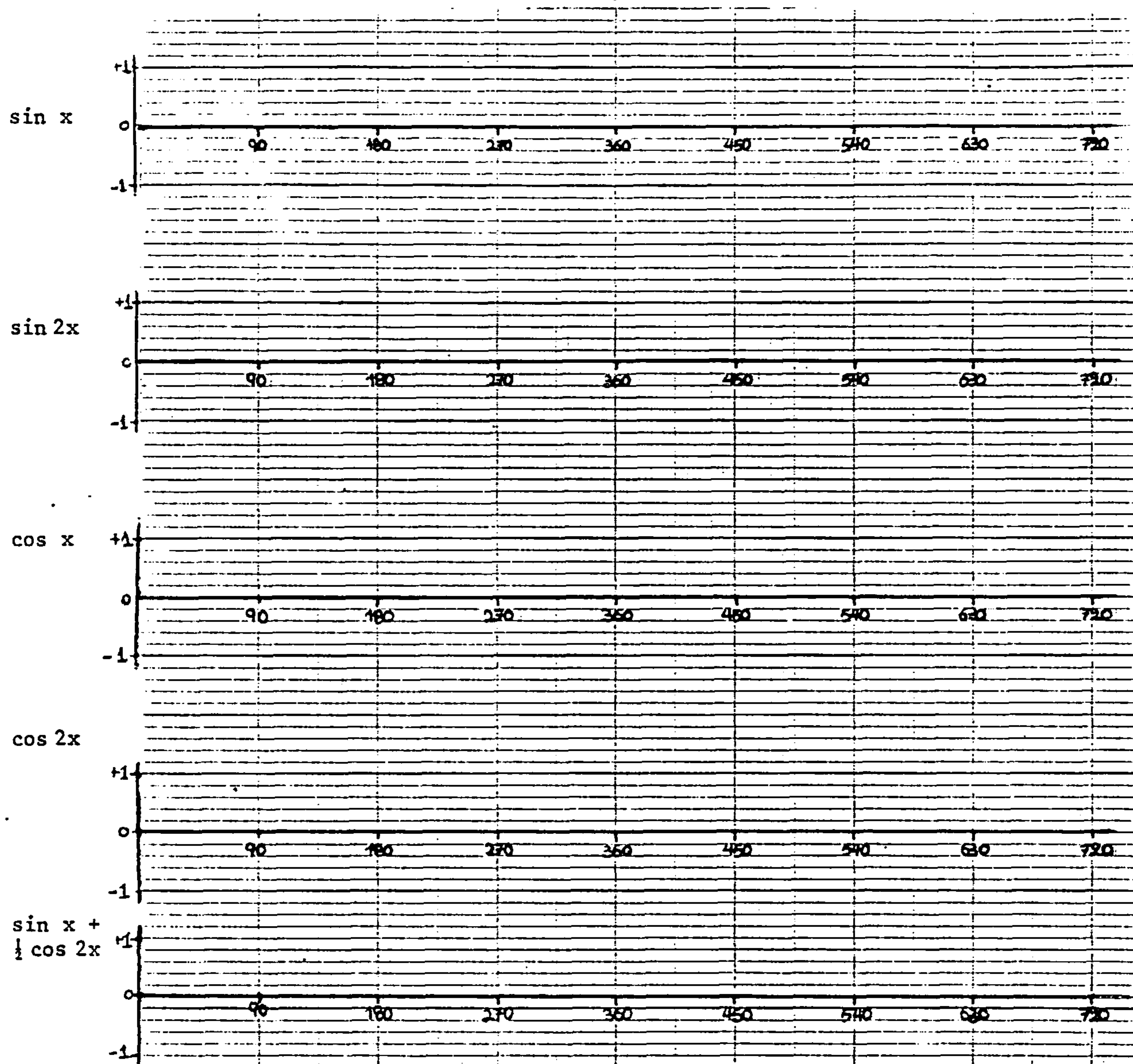
The minimum period of the function $g(x)$ is degrees.



The minimum period of the function $h(x)$ is degrees

2) Please, sketch the graphs of the functions
 $\sin x$, $\sin 2x$, $\cos x$, $\cos 2x$, $\sin x + \frac{1}{2} \cos 2x$ and
 find their minimum period.

Contd./



minimum period of $\sin x$ is degrees
 minimum period of $\sin 2x$ is degrees
 minimum period of $\cos x$ is degrees
 minimum period of $\cos 2x$ is degrees
 minimum period of $\sin x + \frac{1}{2} \cos 2x$ is degrees

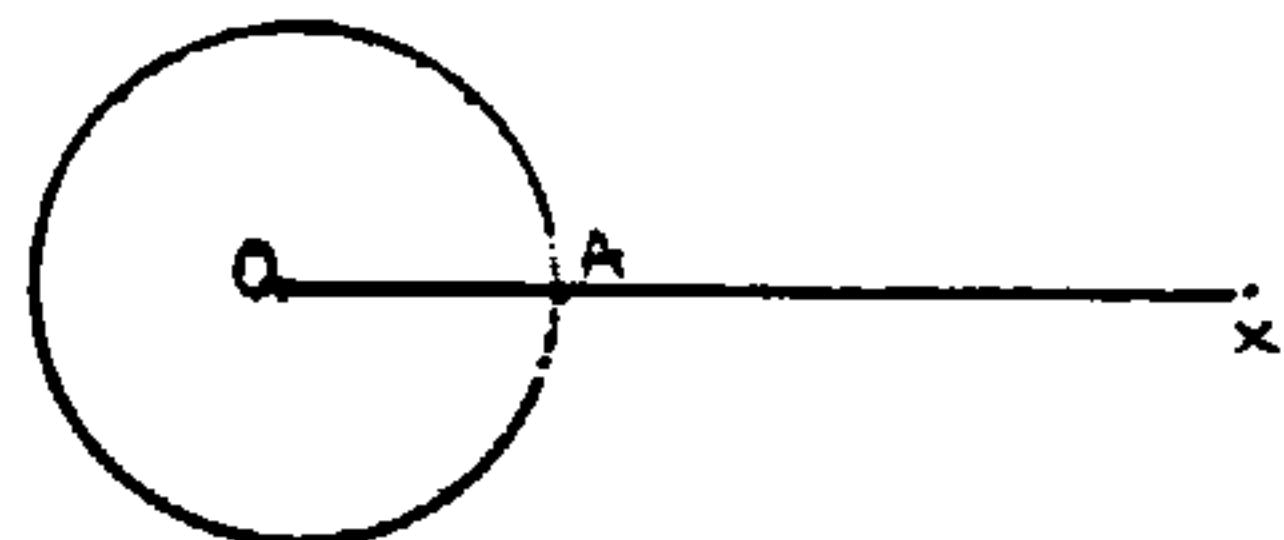
Radian

1. Complete the table below

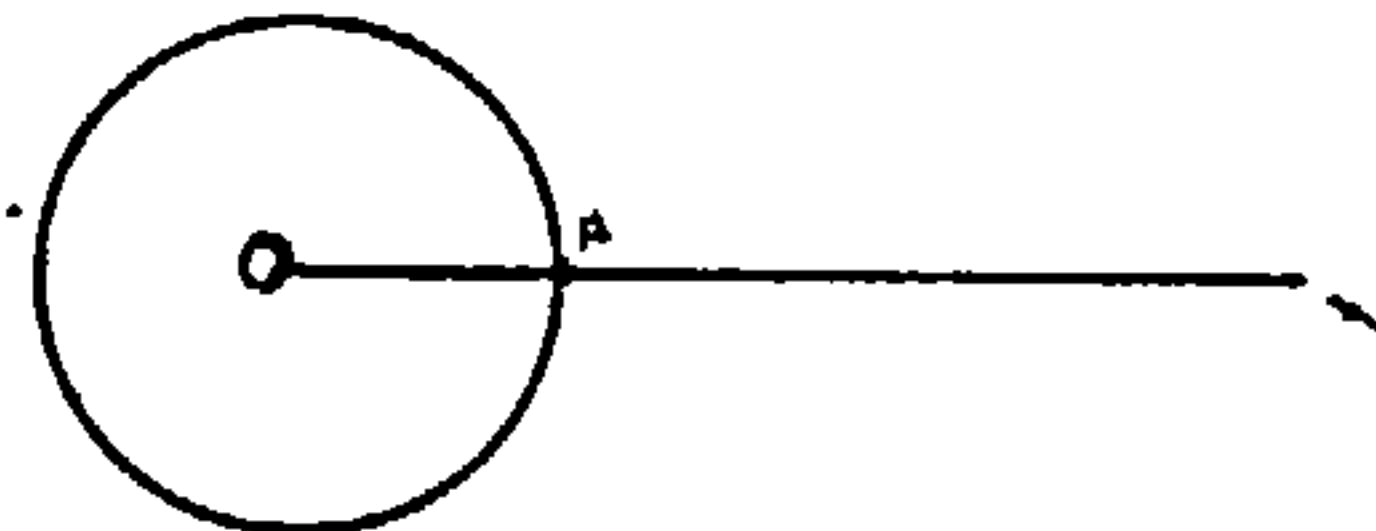
degrees	radians
30	
60	
45	
150	
360	

2. Draw the angle $\angle AOB$, subtended at the centre of a circle of unit radius, with one side Ox and direction as shown by the arrow.

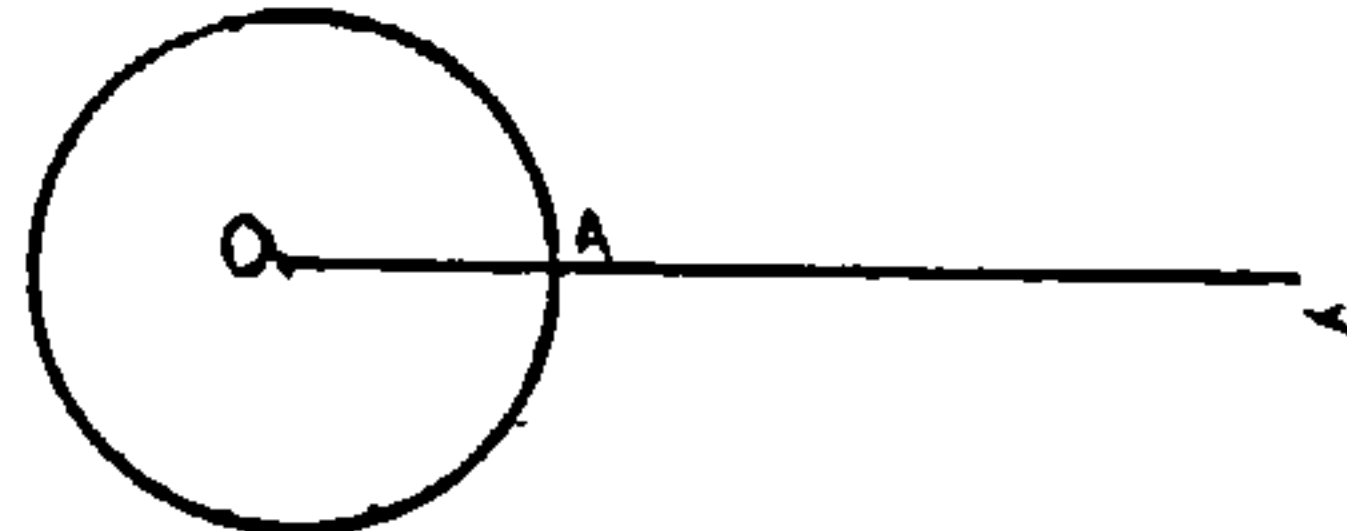
a) $\pi/3$ radians



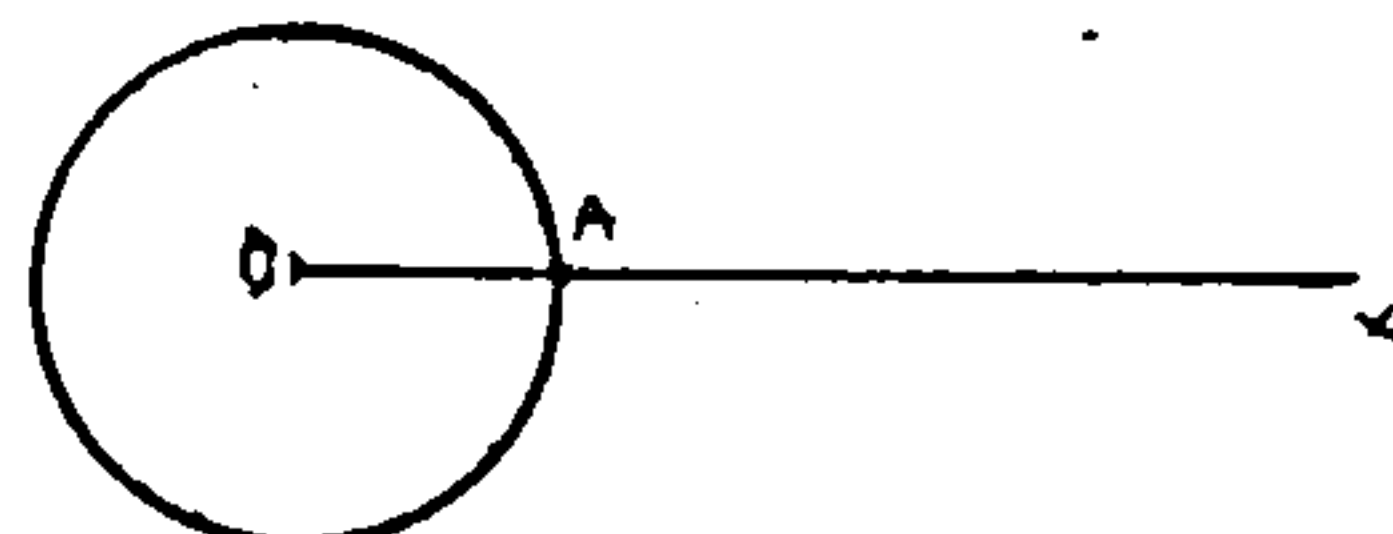
b) $\pi/6$ radians



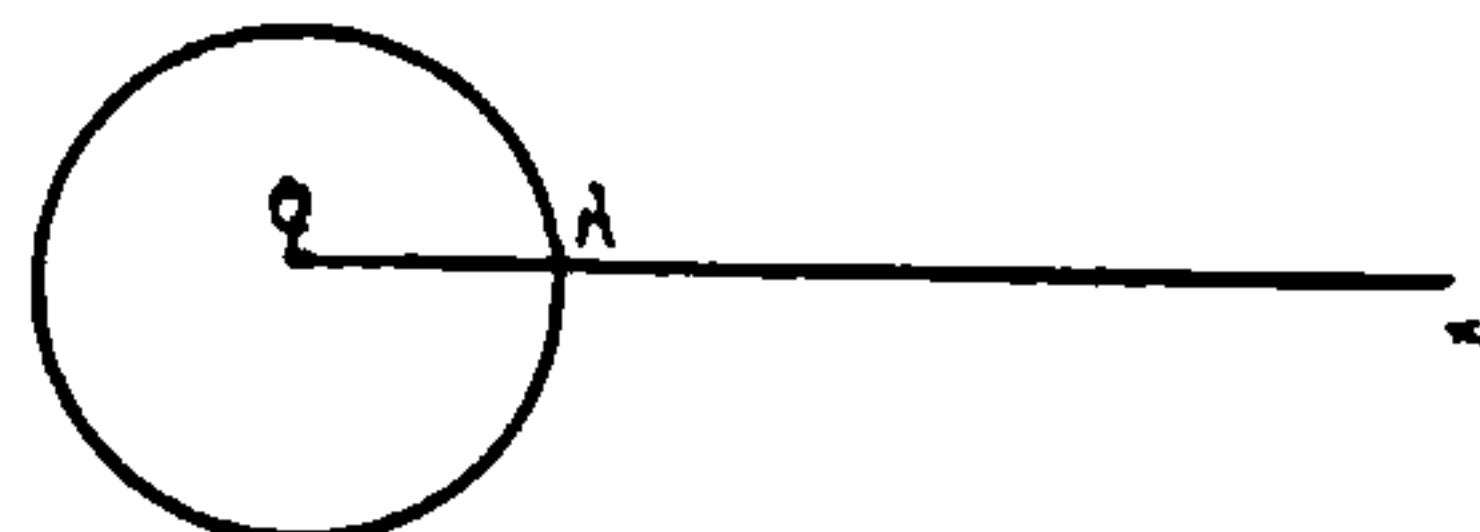
c) $2\pi/3$ radians



d) $\pi/2$ radians

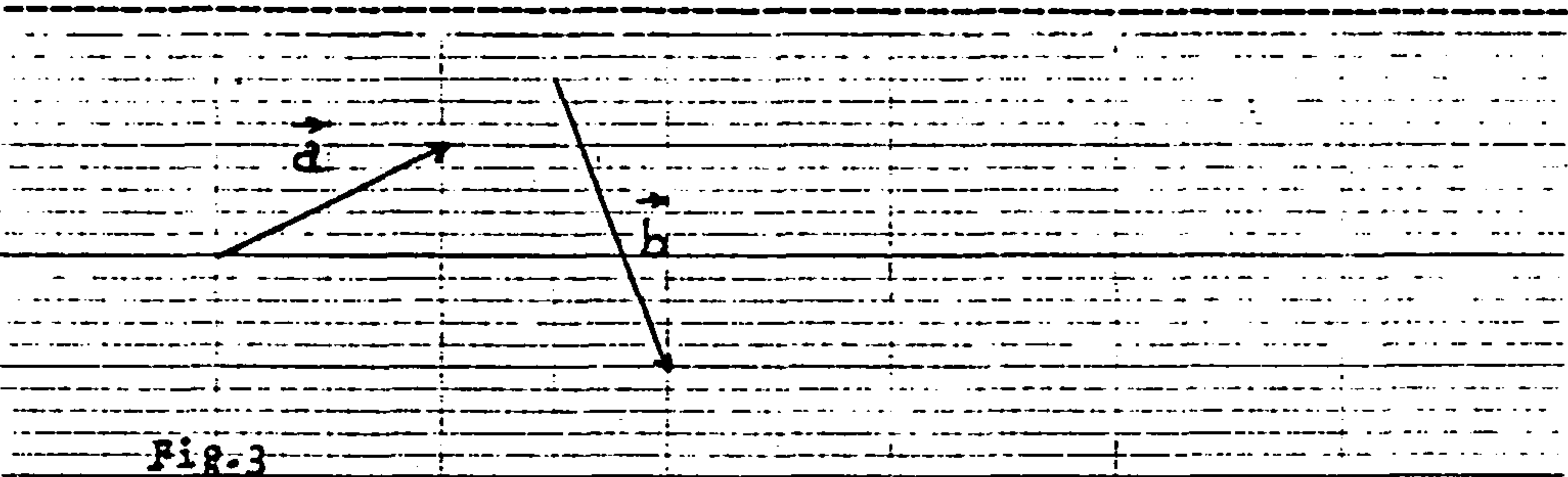
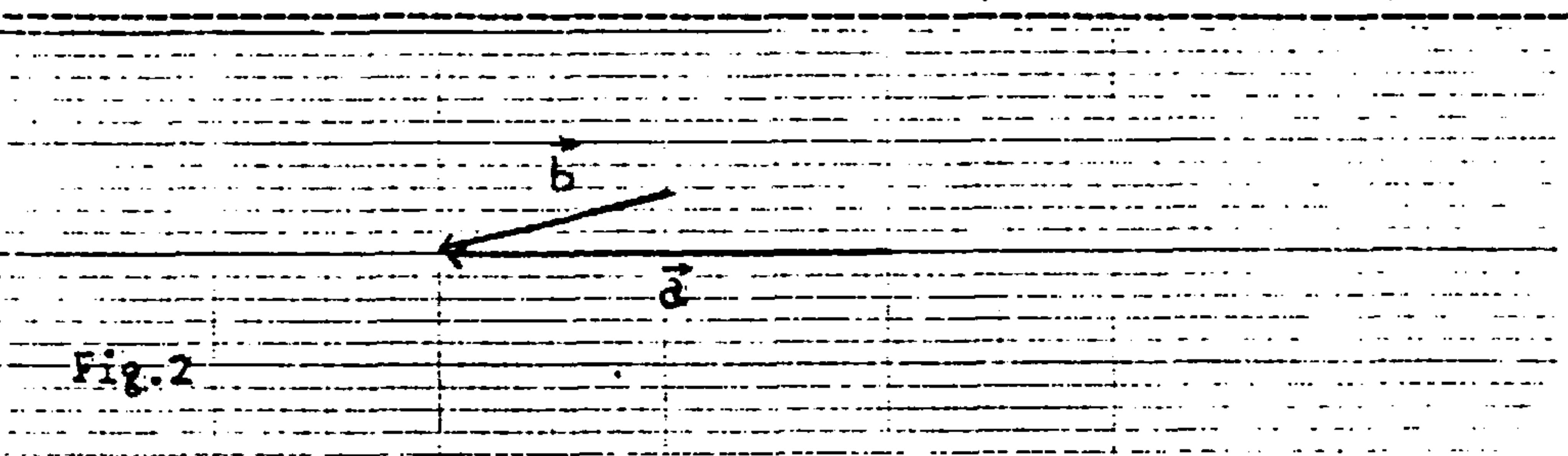
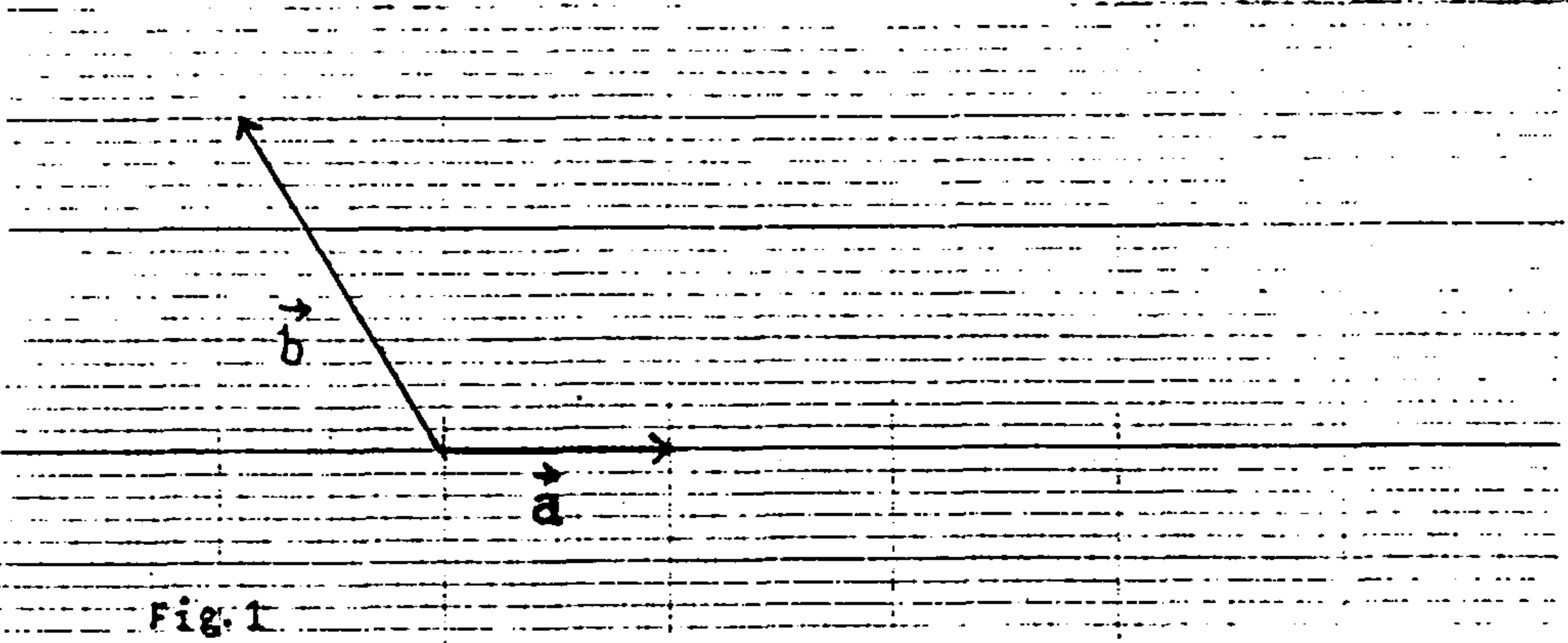


e) π radians



Vectors

- 1) Draw the vector which represents the sum of the two vectors in Fig.1, Fig.2, Fig.3:



- 2) A bird flies due East at 50 mph and a 15 mph wind is blowing from the South West. Use a vector diagram to show the true direction and magnitude of the bird's velocity.

Bridge

*** Please complete this sheet without using the computer.***

1.(a) What is the shape of the cable which supports the bridge?

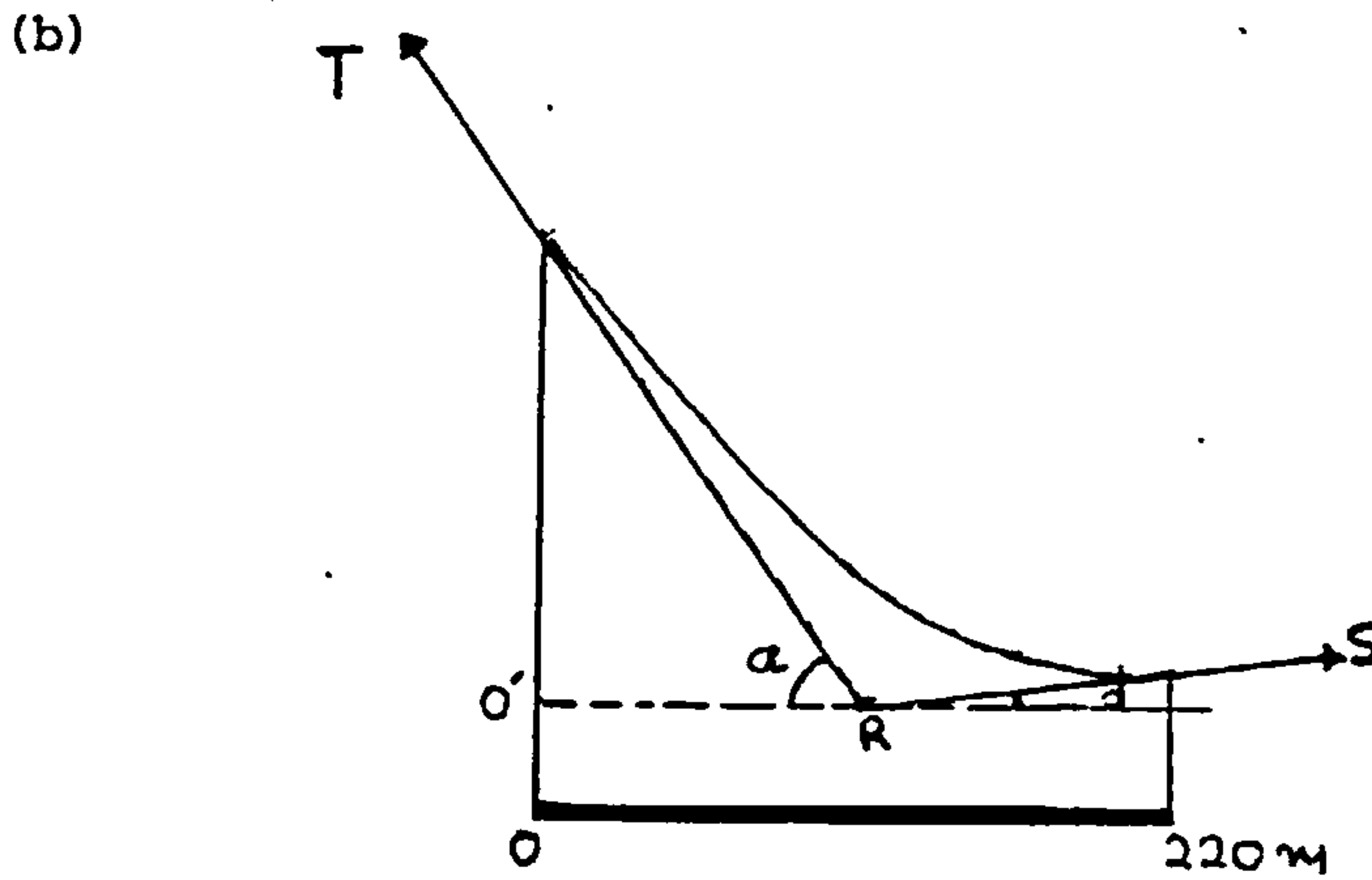


Fig.1

At the point 220 m from the lefthand end of the bridge, the tension in the cable is S Ktn and has the direction shown in the diagram above. The lines of action of the forces T and S meet at the point R.

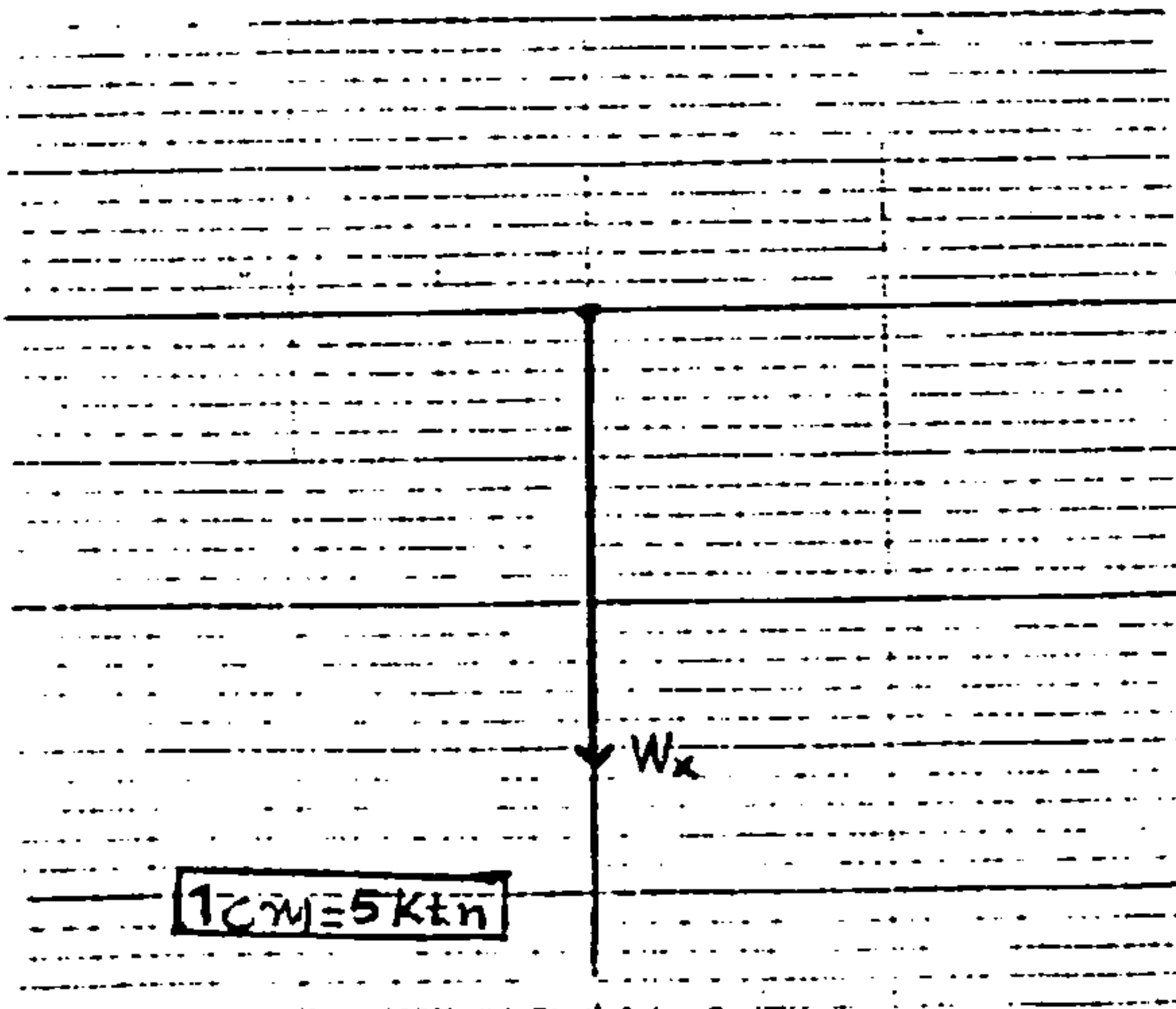
How far is R from the left hand end of the bridge, i.e. what is O'R ? O'R = m.

(c) Write down in terms of S,T, the part weight of the bridge W_x , and the angles a and d the equations for the forces acting on the bridge.

horizontal forces

vertical forces

2.



The weight supported by S and T where $x = 220$ m is $W_x = 15.9$ Ktn.

In the diagram beside, complete the triangle of forces using the directions of T and S in Fig.1.

By measuring the sides of the triangle find the value for T and S.

T = Ktn

S = Ktn.

Evaluation of consolidation and practice exercisesTeacher's questionnaire

(To be completed by the teacher)

1. How many students are there in your class ?

2. How many students used the booklets ?

3. Do you think that these exercises are

very helpful

helpful

not helpful

☐☐☐

to the students' performance ? (Tick (✓) your choice)

4. Can you suggest any other topics for revision which you would like to add to this set of exercises ?

D. Potari

University of Edinburgh

Department of Mathematics

James Clerk Maxwell Buildings,

The King's Buildings,

Mayfield Road,

Edinburgh, EH9 3JZ

* Please return this questionnaire with the students' appraisal forms to me. You may keep the booklets for further use if you wish.

Student's questionnaire

(To be completed by the students)

Daily Refreshers

Performance Record

Set	1	2	3	4	5	6	7	8	9	10
Time taken : (minutes)										
Answers correct first time										

When all ten sets have been done, please complete appraisal below.

1. You found the questions

very easy

easy

OK

hard

very hard

☐☐☐☐☐

Tick (✓) your choice.

2. Do you think doing these questions has helped you with your mathematical studies ?

a lot

a little

not at all

☐☐☐

Tick (✓) your choice.

3. Any comments ?

Please return to your teacher or to Miss Despina Potari, University of Edinburgh
Department of Mathematics, Room 5315,
James Clerk Maxwell Building, King's Building
Mayfield Road, Edinburgh EH9 3JZ.

(To be completed by the students)

Daily RefreshersPerformance Record

Set	11	12	13	14	15	16	17	18	19	20
Time taken : (minutes)										
Answers correct first time										

When all ten sets have been done, please complete appraisal below.

1. You found the questions

very easy

☐

easy

☐

OK

☐

hard

☐

very hard

☐

Tick (✓) your choice.

2. Do you think doing these questions has helped you with your mathematical studies ?

a lot

☐

a little

☐

not at all

☐

Tick (✓) your choice.

3. Any comments ?

Please return to your teacher or to Miss Despina Potari, University of Edinburgh
Department of Mathematics, Room 5315,
James Clerk Maxwell Building, King's Buildings,
Mayfield Road, Edinburgh EH9 3JZ.

(To be completed by the students)

Daily Refreshers

Performance Record

Set-	21	22	23	24	25	26	27	28	29	30
Time taken : (minutes)										
Answers correct first time										

When all ten sets have been done, please complete appraisal below.

1. You found the questions

very easy

easy

OK

hard

very hard

☐

☐

☐

☐

☐

Tick (✓) your choice.

2. Do you think doing these questions has helped you with your mathematical studies ?

a lot

a little

not at all

☐

☐

☐

Tick (✓) your choice.

3. Any comments ?

Please return to your teacher or to Miss Despina Potari, University of Edinburgh
Department of Mathematics, Room 5315,
James Clerk Maxwell Building, King's Buildings,
Mayfield Road, Edinburgh EH9 3JZ.

(To be completed by the students)

Daily Refreshers

Performance Record

Set	31	32	33	34	35	36	37	38	39	40
Time taken : (minutes)										
Answers correct first time										

When all ten sets have been done, please complete appraisal below.

1. You found the questions

very easy

☐

easy

☐

OK

☐

hard

☐

very hard

☐

Tick (✓) your choice.

2. Do you think doing these questions has helped you with your mathematical studies ?

a lot

☐

a little

☐

not at all

☐

Tick (✓) your choice.

3. Any comments ?

Please return to your teacher or to Miss Despina Potari, University of Edinburgh
Department of Mathematics, Room 5315,
James Clerk Maxwell Building, King's Buildings,
Mayfield Road, Edinburgh EH9 3JZ.

APPENDIX C

Evaluation of practical activitiesTeacher's questionnaire

Practical Activity Number

College :

Lecturer:

Date :

No. of students in class

1. Time spent on this activity (approximately) minutes

2. Did the students like the activity ?

3. Do you think that the activity is worth the time spent on it ?

4. Was there any problem either with the equipment or with the instructions given ?
If 'Yes' please specify :

5. Please describe any other problems which caused difficulties to you or to the students during the work in this activity in the classroom.

6. Do you have any suggestion for improvement of the activity ? Please feel free to add any other comments you may have.

Student's questionnaire

Appraisal

Practical Activity Number

☐

(Please return to Miss D. Potari)

Name : _____

College: _____

Date : _____

Please answer the questions below

1. Time spent on this activity (approximately) ☐ minutes

2. Did you find this activity

not interesting at all

☐

interesting

☐

very interesting

☐

Tick your option

3. Did you have any problem understanding the instructions given in the workcard ? If 'Yes', please specify.

4. Did you have any problem in using the equipment ? If 'Yes' please specify.

5. Please write below any suggestions you may have about the improvement of this activity.

6. Further comments ?

Teacher's instructions for the
investigation competition
Investigations Competition 1986-7

Four investigations have been chosen for this small competition.

These are :

1. The Best Way of Calculating the Square Root of Two.
2. Design a Two-Pint Milk Carton.
3. A Series of Equilateral Triangles.
4. Working out the Values of $\sin\theta^\circ$ ($0 \leq \theta \leq 90$) .

These investigations have been chosen because they are suitable for a wide range of ability.

Investigation 2 (Design a two-pint milk carton) is described in the Guidelines for Teaching and Learning Approaches for the 16-18 Action Plan., Module M2/A1. A possible approach is described in the guideline.

In investigation 3 (A Series of Equilateral Triangles) some book references have been added to help the students with their investigation of properties like focus of a parabola, tangent, latus rectum, vertex, axis of symmetry.

In investigation 4 a generalisation of the method used for calculating $\sin 32.4^\circ$ is asked for which can be used for calculating $\sin\theta^\circ$ for $0 < \theta < 90$.

Some possible approaches or comments to the investigations 1,3 and 4 are given in the teaching guide.

It is expected that the students will work on these investigations in their own time attempting one investigation every fortnight. Please give an introduction to each one.

It is expected that students will have difficulty with this method of working in Mathematics, although many will have used it for statistics and computing.

Please give them some general suggestions about the investigation, emphasising that there is not a unique way of approaching the problem nor a unique answer.

The students may ask for advice and help during their work on the investigation (for example you might arrange a time when they can come for advice and help). They will need encouragement to complete their task: Each essay must be at most 1000 words. (i.e. 4 sides of A4). The presentation of their work is very important.

The students should be encouraged to write clearly, give references to sources of information, draw carefully any graphs, set out clearly numerical results. When they have written computer programs for the solution of the problems, the computer programs and the results should be documented. If you have the time, we would be grateful for your comments on their work and for your ranking of the essays. Please send the essays to Miss Despina Potari, Department of Mathematics, University of Edinburgh, James Clerk Maxwell Building, King's Buildings, Mayfield Road, Edinburgh EH9 3JZ. We will correct them and return them to you as soon as possible. It would be very helpful to have some comments from the students about the exercises, how long they spent on the exercise, whether they enjoyed it, whether they found it difficult or easy, etc... This information would be separated from the essays and would not affect the award of the prize.

It would be also very helpful for us to have any comments you may have about the work of the students through the investigations (It was very difficult for their level, they found it interesting, boring, etc...)

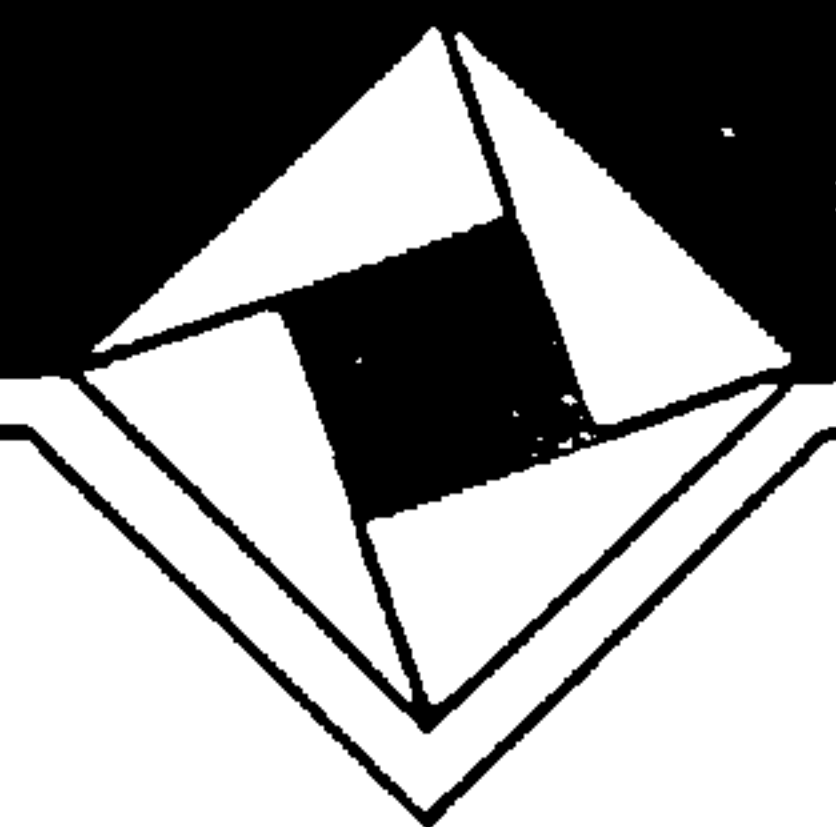
Thank you for your assistance with this work.

Despina Potari.

Final questionnaires
Teacher's questionnaire

EDINBURGH MATHEMATICS

Teaching Group



University of Edinburgh
Department of Mathematics
James Clerk Maxwell Building, The King's Buildings, Mayfield Road, Edinburgh, EH9 3JZ

Module M2/A1 : Course Materials Appraisal

School/College.....

1. Number of students in the class
2. Is there enough equipment available in your classroom to use these materials (eg. microcomputers, overhead projectors etc...)?
3. In a few words, describe your relationship with the students (eg. -good contact, no communication etc..)
4. Similarly, describe the relationship between the students in the classroom (eg. cooperative, they work together, etc...)
5. In which ways did you help the students to learn (eg. lecturing, discussing with them individually, suggesting additional work, etc...)?
6. Of the materials which I sent you which ones did you actually use in the course? (Tick (✓) them) .

computer programs	<input type="checkbox"/>
practical activities	<input type="checkbox"/>
consolidation and practice exercises	<input type="checkbox"/>
posters	<input type="checkbox"/>
video taped expositions	<input type="checkbox"/>
investigations	<input type="checkbox"/>
tape booklets	<input type="checkbox"/>

7. On average how much time did you spend on the materials per week (i.e. per 4 hours) ?

Time spent

computer programs	
practical activities	
video taped (or teacher's) expositions, including discussion and activities of the students based on the programmes	
investigations	
tape booklets (if they were used in the classroom)	

8. Describe any problems which you had in using the materials or which prevented you from using them. (eg. not enough availability of computers, the materials did not work properly etc...)

Problems

Computer programs :

Practical activities :

Consolidation and practice exercises :

Posters :

Videotaped expositions:

Investigations :

Tape booklets :

9. Other questions about the materials :

Computer programs :

- a) How were the programs used? (eg. illustrative, reinforcement materials etc..)
- b) Describe the students' reactions to them (eg. they liked, they found them boring, etc...)

Practical activities :

- a) How were they used? (eg. work in groups, discussion about the results etc..)
- b) Describe the students' reactions to them (eg. they liked or not etc...)

Posters

- a) Did the students notice them? (by themselves or after your encouragement?)
- b) What effects did they have in the classroom? (eg. discussion or a possible investigation arising from them etc...)
- c) Describe the students' reactions to them (eg. they liked or not etc...)

Videotaped expositions :

- a) How were they used? (eg. discussion after each program, additional teaching, practice by the students etc...)
- b) Describe the students' reactions to them (eg. they liked or not etc...)

Investigations :

- a) How were they used? (eg. the students worked together etc...)
- b) Describe the students' reactions to them (eg. they liked or not etc...)

Tape booklets :

- a) Where they were used? (eg. in the college's library, classroom, home etc...)
- b) Describe the students' reactions to them (eg. they liked or not etc.)

- 10. What other materials were used? (eg. books, notes etc...)
- 11. Which materials do you think were most successful in the classroom?
- 12. Did the organisation of the college cause some problems in the course? (eg. classes, equipment, timetable etc..)

13. How much do you think the students found the course :
- enjoyable?
 - difficult?
 - worthwhile?
 - anything else?
14. Did you have any problems in teaching the course because of the characteristics of this particular class? (eg. lack of uniform background, ages etc...)
15. How was the course assessed? Would you like to assess any parts of it in other ways?
16. Which of the materials you have been given would you like to use in the future?
17. Are there other materials you would also like to use in the future?
18. Apart from the materials I have sent you, did you use teaching materials obtained from any other source?
- How do you think new ideas and materials could best be disseminated?
19. What kind of support would you like to have to make the course more successful? (eg. from the college or from different authorities etc...)
20. Have you any other suggestions?

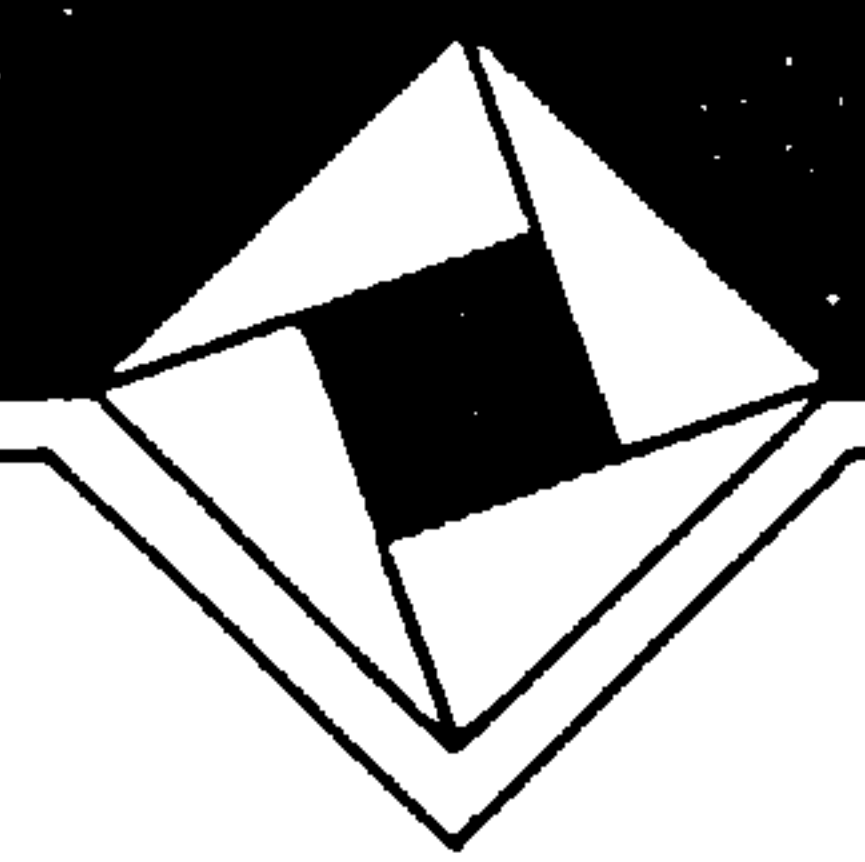
Thank you very much for the time you spent completing this appraisal and for your cooperation and help in developing these teaching materials.

D. Potari

Student's questionnaire

EDINBURGH MATHEMATICS

Teaching Group



University of Edinburgh
Department of Mathematics
James Clerk Maxwell Building, The King's Buildings, Mayfield Road, Edinburgh, EH9 3JZ

Module M2/A1 : Course Appraisal

School/College.....

1. Did you like the course? Why?
2. Do you think the course will be useful for your future studies?
3. Have you noticed any improvement in your ability in mathematics as a result of the course?
4. Do you like mathematics now more than before starting the course or less?
5. Describe your relationship with the other students and with your teacher (eg. friendly, cool etc.).

6. Please identify which topics of the course you found difficult

ERRORS	Very difficult	<input type="checkbox"/>
	difficult	<input type="checkbox"/>
	average	<input type="checkbox"/>
	easy	<input type="checkbox"/>
	very easy	<input type="checkbox"/>

LINEAR FUNCTION	very difficult	<input type="checkbox"/>
	difficult	<input type="checkbox"/>
	average	<input type="checkbox"/>
	easy	<input type="checkbox"/>
	very easy	<input type="checkbox"/>

QUADRATIC FUNCTION	very difficult	<input type="checkbox"/>
	difficult	<input type="checkbox"/>
	average	<input type="checkbox"/>
	easy	<input type="checkbox"/>
	very easy	<input type="checkbox"/>
TRIGONOMETRY	very difficult	<input type="checkbox"/>
	difficult	<input type="checkbox"/>
	average	<input type="checkbox"/>
	easy	<input type="checkbox"/>
	very easy	<input type="checkbox"/>
VECTORS	very difficult	<input type="checkbox"/>
	difficult	<input type="checkbox"/>
	average	<input type="checkbox"/>
	easy	<input type="checkbox"/>
	very easy	<input type="checkbox"/>

If you have particular difficulties with any of those topics, can you explain what caused them?

7. Please tick which of the materials below you used

Computer programs	<input type="checkbox"/>
practical activities	<input type="checkbox"/>
consolidation and practice exercises	<input type="checkbox"/>
posters	<input type="checkbox"/>
videotaped expositions	<input type="checkbox"/>
investigations	<input type="checkbox"/>
tape booklets	<input type="checkbox"/>
books (specify)	<input type="checkbox"/>
notes	<input type="checkbox"/>
others (specify)	<input type="checkbox"/>

8. Did you find the

<u>Computer programs</u>	very interesting	<input type="checkbox"/>	Tick (✓) your option.
	interesting		
	not interesting at all	<input type="checkbox"/>	
<u>practical activities</u>	very interesting	<input type="checkbox"/>	Tick (✓) your option.
	interesting		
	not interesting at all	<input type="checkbox"/>	

<u>consolidation and practice exercises</u>	very interesting interesting not interesting at all	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Tick (✓) your option.
<u>posters</u>	very interesting interesting not interesting at all	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Tick (✓) your option.
<u>videotaped expositions</u>	very interesting interesting not interesting at all	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Tick (✓) your option.
<u>investigations</u>	very interesting interesting not interesting at all	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Tick (✓) your option.
<u>tape booklets</u>	very interesting interesting not interesting at all	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Tick (✓) your option.
<u>books</u>	very interesting interesting not interesting at all	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Tick (✓) your option.
<u>notes</u>	very interesting interesting not interesting at all	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Tick (✓) your option.
<u>others</u>	very interesting interesting not interesting at all	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Tick (✓) your option.

9. Did you find the

<u>computer programs</u>	very helpful helpful not helpful at all	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Tick (✓) your option.
<u>practical activities</u>	very helpful helpful not helpful at all	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Tick (✓) your option.
<u>consolidation and practice exercises</u>	very helpful helpful not helpful at all	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Tick (✓) your option.

<u>posters</u>	very helpful helpful not helpful at all	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Tick (✓) your option
<u>videotaped expositions</u>	very helpful helpful not helpful at all	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Tick (✓) your option
<u>investigations</u>	very helpful helpful not helpful at all	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Tick (✓) your option
<u>tape booklets</u>	very helpful helpful not helpful at all	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Tick (✓) your option
<u>books</u>	very helpful helpful not helpful at all	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Tick (✓) your option
<u>notes</u>	very helpful helpful not helpful at all	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Tick (✓) your option
<u>others</u>	very helpful helpful not helpful at all	<input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/>	Tick (✓) your option

10. What qualifications in mathematics did you have before you started the course? (eg. O grade etc.)
11. Are there any problems you faced with the course that you would like to comment on?
12. What sort of changes would you like to see in the course? (eg. more self-help materials, better learning environment,...)
13. Can you make any other suggestions for improving the course?

14. Are there any other comments, about the course, the materials, the college or anything else, that you would like to make?

Thank you very much for your help and cooperation.

D. Potari